Introduction to Quantitative Methods

NQF

Subject Examiner's Report

Unit Title:	**Introduction to Quantitative Methods**
Unit Code:	**1.4 IQM**
Session:	June 2016

Question 1

(a) Without the use of a calculator, express each answer as a fraction in its simplest form. (You must show all steps in your calculations).

(i) $\sqrt{\frac{4}{9}} \div \left(\frac{2}{4}\right)^2$ **(4 marks)**

(ii) $\left(\frac{8}{16} \times \frac{10}{2}\right) \div 1^{0.5}$ **(4 marks)**

(iii) $\frac{-4(2-6)+(-6)-(-3)}{-4+30}$ **(4 marks)**

(b) A cocoa trading company generated sales revenue of £845,642 in the 2015/16 financial year.

(i) **Express this sales revenue correct to 3 significant figures.** **(2 marks)**

(ii) **Express this sales revenue in standard form $A \times 10^n$ (where $1 \leq A < 10$ and n is an integer).** **(2 marks)**

(iii) **Calculate the value of this sales revenue in Botswana Pula (BWP), using an exchange rate of £1 = 15.52 BWP.** **(2 marks)**

(iv) **Calculate the percentage increase in profit between 2014/15 and 2015/16, assuming that the sales revenue of the company was £735,341 in the financial year 2014/2015.** **(2 marks)**

(c) Express 0.125:

(i) **correct to 2 decimal places** **(1 mark)**

(ii) **as a fraction in its simplest form** **(2 marks)**

(iii) **as a percentage** **(2 marks)**

1. Comments on learners' performance

Part (a) required the application of the four rules of numeracy to fractions, including the use of square roots. While most students performed very well in part (i), a number of students did not understand how to square or divide fractions. In part (ii) students also performed well, although a number of students showed difficulty solving $1^{0.5}$. Part (iii) was answered less well, with many students failing to carry out the arithmetic operations in the correct sequence, failing to understand the implication of the brackets and failing to understand the implication of the minus signs. Common to parts (i) to (iii), a minority of students failed to show all relevant steps in their calculations and/or failed to express each answer as a fraction in its simplest form as stipulated in the question.

Parts (b) and (c) required students to express data in different forms. Compared with previous examination sessions, many students showed difficulty calculating the percentage increase in revenue between the two years.

2. Mark scheme

(a) To gain full marks, worked steps must be shown and each answer must be shown as a fraction in its simplest form. Where the fraction is not expressed in its simplest form, award a maximum of 3 marks for each fraction given. The use of a calculator is not permitted. Award 1 mark for each correct answer where no workings are shown. Where the final answer is incorrect but the correct method has been used, award up to 2 method marks for each part.

(i) $\sqrt{\frac{4}{9}} \div \left(\frac{2}{4}\right)^2 \quad = \frac{2}{3} \div \frac{4}{16} \quad = \frac{2}{3} \times \frac{16}{4} \quad = \frac{32}{12} \quad = \frac{8}{3}$ or $2\frac{2}{3}$

(ii) $\left(\frac{8}{16} \div \frac{2}{10}\right) \div 1^{0.5} \quad = \left(\frac{8}{16} \times \frac{10}{2}\right) \div \sqrt{1} \quad = \left(\frac{80}{32}\right) \div 1 \quad = \frac{80}{32} \quad = \frac{5}{2}$ or $2\frac{1}{2}$

(iii) $\frac{-4(2-6)+(-6)-(-3)}{-4+30} \quad = \frac{(-8+24)+(-3)}{26} \quad = \frac{16-3}{26} \quad = \frac{13}{26} \quad = \frac{1}{2}$

(b) Award 2 marks for each correct answer and 1 method mark where the final answer is incorrect but there is evidence that the correct method has been used.

(i) £846,000
(ii) 8.45642×10^5
(iii) £845,642 × 15.52 = BWP 1,312,436.84
(iv) $\frac{845{,}642 - 735{,}341}{735{,}341} \times \frac{100}{1} = 15\%$

(c) Award 1 mark for part (i) and 2 marks for each correct answer to parts (ii) and (iii). Award 1 method mark for parts (ii) and (iii) where the final answer is incorrect but there is evidence that the correct method has been used.

(i) 0.13
(ii) $\frac{125}{1000} \quad = \frac{1}{8}$
(iii) $\frac{1}{8} \times \frac{100}{1} \quad = 12.5\%$

3. Recommendations

Students and tutors are advised to spend a little more time applying the four rules of numeracy without the use of a calculator, focusing in particular on the division of fractions, the subtraction of negative numbers and the importance of correctly sequencing arithmetic operations. In addition, students and tutors should spend a little more time learning how to calculate percentages.

Examiner's tips

Do not forget to show all relevant steps in your calculations and when stipulated in the question, always express fractions in their simplest form.

Question 2

(a) An investor deposits £20,000 in a bank account that pays 6% compound interest per annum. Calculate how much interest the investor will earn after 6 years. (Give your answer to the nearest £.) **(5 marks)**

(b) Calculate the annual rate of compound interest that would be necessary for a £20,000 investment to grow to £30,000 by the end of 6 years. (Give your answer correct to 1 decimal place.) **(5 marks)**

(c) A company purchased a machine for £20,000. Calculate the value of the machine after four years if it is depreciated by:

(i) £2,000 per year using the straight line method **(5 marks)**

(ii) 10% per year using the reducing balance method **(5 marks)**

(d) Simplify the following logarithm equation to a single log term: log (*x*) + log (*x* − 12) **(5 marks)**

1. Comments on learners' performance

This question concerned the application of quantitative methods to business situations, notably making use of calculations concerning interest and depreciation. In general, this question was relatively well answered compared with previous examination sessions, with a relatively high number of students attaining good marks.

Part (a) required students to calculate the total interest received from an investment at the end of a six year period, with compound interest applied. Students performed well on this question although a number of students calculated the interest earned using the simple (instead of the compound) interest formula. Although the majority of students did use the correct (compound interest) formula to calculate the value of the interest earned, a very common error made by students was to forget to subtract the initial invested sum to calculate just the total value of the interest earned over the period (as specified in the question).

Part (b) required students to rearrange the formula for calculating compound interest in order to solve for the interest rate of an investment, given that the original principal, the accrued amount and the term were known. While there was a marked improvement in the number of students that were able to accurately demonstrate how to rearrange the formula and solve for the interest rate, compared with previous examination sessions, there were still a number of students that were unable to complete all of the mathematical steps required.

Part (c) required students to apply both the straight line and reducing balance methods to calculate the value of an asset. There are still a number of students that do not understand how to apply the straight line method correctly, with a level of confusion over whether the calculated value relates to the depreciated asset value or the total level of depreciation. In contrast, students performed better at calculating the value of an asset using the reducing balance method.

Part (d) required students to simplify a logarithm equation to a single log term. Compared to previous examinations, fewer students were able to simplify this logarithm correctly; often unaware of which logarithm rules to use. Nevertheless, a good number of students made a very good attempt at applying the appropriate logarithm rules to simplify the equation to a single log term.

2. Mark scheme

(a) Award 5 marks for a correct answer and up to 3 marks where the final answer is incorrect but there is evidence that the correct method has been used. If the final answer is not given to the nearest £, then award a maximum of 4 marks.

$$20{,}000 \times \left(1 + \frac{6.0}{100}\right)^6 - 20{,}000 \quad = 20{,}000 \times 1.419 - 20{,}000$$

$$= 28{,}370 - 20{,}000 \quad = £8{,}370$$

(b) Award 5 marks for a correct answer and up to 3 marks where the final answer is incorrect but there is evidence that the correct method has been used. If the final answer is not given to 1 decimal place, then award a maximum of 4 marks.

$$20{,}000 \times \left(1 + \frac{i}{100}\right)^6 = 35{,}000 \qquad \therefore \left(1 + \frac{i}{100}\right)^6 = \frac{35{,}000}{20{,}000} \qquad \therefore \left(1 + \frac{i}{100}\right)^6 = 1.5$$

$$\therefore 1 + \frac{i}{100} = 1.5^{\frac{1}{6}} \qquad \therefore 1 + \frac{i}{100} = 1.069913 \qquad \therefore i = (1.069913 - 1) \times 100$$

$$\therefore i = 7.0\%$$

(c) Award 5 marks for a correct answer to each part and up to 3 marks each where the final answer is incorrect but there is evidence that the correct method has been used.

(i)

Initial cost:		= £20,000
Depreciation:	£2,000 × 4	= £8,000
Value of machine after four years:	£20,000 – £8,000	= £12,000

(ii) The value at the end of each year is 90% of the value at the beginning of the year:

Value at the end of year 1:	£20,000 × 90%	= £18,000
Value at the end of year 2:	£18,000 × 90%	= £16,200
Value at the end of year 3:	£16,200 × 90%	= £14,580
Value at the end of year 4:	£14,580 × 90%	= £13,122

(d) Award 5 marks for a correct answer and up to 3 marks each where the final answer is incorrect but there is evidence that the correct method has been used.

Using the rule: $\log (p \times q) = \log p + \log q$

$\log (x) + \log (x - 12) \quad = \log (x (x - 12)) \quad = \log (x^2 - 12x)$

3. Recommendations

Students and tutors are advised to spend a little more time understanding how to use different formulae for different business applications, notably the use of the compound interest formula to calculate the time required for a given investment to accrue a given level of interest, and the use of the straight line method to calculate the depreciated value of an asset. In addition, students and tutors should spend a little more time simplifying logarithm expressions to a single log term, with a particular focus on choice and application of the appropriate logarithm rules.

Examiner's tips

Always try to visualise the mathematical problem being examined. You should be able to tell if your answer is likely to be correct even without knowing what the answer should be.

Question 3

(a) Solve the following equations:

(i) $10x - 3 = 33 + x$ **(4 marks)**

(ii) $2x^2 - 5x + 3 = 0$, using factorisation **(4 marks)**

(iii) $x^2 + 3x + 1 = 0$, using the quadratic formula. (Give your answer correct to 2 decimal places.) **(5 marks)**

(b) For each of the following straight lines, find the values of *m* and *c* to identify the linear equation in the form $y = mx + c$:

(i) Line A passes through the point (5,8), with an intercept on the *y*-axis of 3 **(3 marks)**

(ii) Line B passes through the point (−4,0), with a gradient of 2 **(3 marks)**

(iii) Line C passes through the points (−2,4) and (6,8) **(6 marks)**

1. Comments on learners' performance

This question concerned the use of algebraic methods. Generally students performed better answering part (a) than part (b).

Part (a) required students to solve three equations. Students generally performed very well solving the equation in part (i). Compared to previous examination sessions students tended to show more difficulty solving the quadratic equations in parts (ii) and (iii); common mistakes included to deriving the correct factors when using factorisation and ignoring the implication of the minus signs when using the quadratic formula.

Part (b) required students to find the values of *m* and *c* to identify the linear equation in the form $y = mx + c$. Students made a good attempt at this question, with a good number of students demonstrating a clear understanding that *m* represents the gradient and *c* represents the intercept with the *y*-axis. Those students that performed less well on this question generally made errors when applying the formula to solve for *m* and *c*; a very common mistake was to ignore the implication of the minus signs.

2. Mark scheme

(a) Award full marks for each correct answer. Award method marks (up to 2 marks for parts (i) and (ii) and up to 3 marks for part (iii)), where the final answer is incorrect but there is evidence that the correct method has been used. If the quadratic formula has been used instead of factorisation in part (ii), and factorisation instead of the quadratic formula in part (iii), then award up to a maximum of 2 marks for the correct answer.

(i) $10x - 3 = 33 + x$ $\quad \therefore 10x - x = 33 + 3$ $\quad \therefore 9x = 36$ $\quad \therefore x = 4$

(ii) $2x^2 - 5x + 3 = 0$, using factorisation gives: $\quad (2x - 3)(x - 1) = 0$

either: $\quad 2x - 3 = 0$ or $x - 1 = 0$

so: $\quad x = 1.5$ or $x = 1$

(iii) $x^2 + 3x + 1 = 0$, using the quadratic formula gives:

$$x = \frac{-3 \pm \sqrt{3^2 - (4 \times 1 \times 1)}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{9 - 4}}{2}$$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

$$x = \frac{-3 \pm 2.236}{2}$$

so: $x = \frac{-0.764}{2}$ or $x = \frac{-5.236}{2}$

x = -0.38 or x = –2.62

(b) Award full marks for a correct answer to each part. Award up to 2 method marks each for parts (i) and (ii), and up to 4 method marks for part (iii), where the final answer is incorrect but there is evidence that the correct method has been used.

(i) Line A: $x = 5$, $y = 8$ and $c = 3$

If $y = mx + c$, *then:* $8 = (m \times 5) + 3$

$-(m \times 5) = 3 - 8$

$-m = (3 - 8) \div 5 \Rightarrow m = 1$

As $m = 1$ and $c = 3$, then $y = 1x + 3$

(ii) Line B: $x = -4$, $y = 0$ and $m = 2$

If $y = mx + c$, *then:* $0 = (2 \times (-4)) + c$

$-c = (2 \times (-4)) - 0$

$-c = (-8) - 0 \Rightarrow c = 8$

As $m = 2$ and $c = 8$, then $y = 2x + 8$

(iii) Line C: $x = -2$, $y = 4$ and $x = 6$, $y = 8$

$$m = \frac{8-4}{6-(-2)}$$

$$m = \frac{4}{8} \Rightarrow m = 0.5$$

If $y = mx + c$, then: $4 = (0.5 \times (-2)) + c$

$-c = (0.5 \times (-2)) - 4$

$-c = (-1) - 4 \Rightarrow c = 5$

As $m = 0.5$ and $c = 5$, then $y = 0.5x + 5$

3. Recommendations

Students and tutors are advised to spend a little more time practicing the application of algebraic methods, focusing in particular on re-arranging formulae and understanding the implication of minus signs.

Examiner's tips

Always consider the implication of minus signs when re-arranging formulae.

Question 4

(a) Explain the circumstances in which a scatter diagram would be the most appropriate form of representing data graphically. (6 marks)

(b) The following table presents data on the number of ice cream sales and air temperature over a 15-day period in the summer of 2015:

Air temperature (°C)	Number of ice cream sales (‘000)
16	1.2
14	1.0
13	0.2
24	2.2
32	3.6
15	1.1
19	1.3
14	0.4
27	2.9
21	1.9
28	3.4
19	1.5
22	1.9
14	0.6
32	3.6

(i) **Draw a fully labelled scatter diagram of air temperature (on the *x*-axis) against ice cream sales (on the *y*-axis). (Use the graph paper at the front of your answer book.) (9 marks)**

(ii) **Using your scatter diagram drawn in (i), comment on the possible relationship between air temperature and ice cream sales. (4 marks)**

(c) The demand function for ice cream (calculated from the data at air temperatures of over 15°C presented in (b)) is given by the equation $y = 0.1613x - 1.5019$, where *y* is number of ice cream sales (‘000) and *x* is the air temperature (°C).

Using this equation, calculate the:

(i) **Number of daily ice cream sales when the air temperature is 25°C. (3 marks)**

(ii) **Air temperature at which 3,000 ice creams would be sold per day. (3 marks)**

1. Comments on learners’ performance

This question concerned the conceptualisation of a business problem through the construction and use of a scatter diagram, and algebraic methods. In general, this question was relatively well answered.

In parts (a) and (b), students tended to perform better drawing their scatter diagram than they did on its interpretation. Relatively few students demonstrated a clear understanding of the circumstances in which a scatter diagram would be the most appropriate form of representing data graphically (part (a)). In the main, students made a very good attempt at plotting their scatter diagram in part (b). Consistent with previous examination sessions, students typically lost marks for: not including a title; not labelling the *x* and *y* axes; and plotting sales revenue on the *x*-axis instead of the *y*-axis. Relatively few students demonstrated a clear understanding of the possible relationship between temperature and sales.

Compared to previous examination sessions, students performed relatively well in part (c) evaluating the demand equation to solve for values of *x* and *y*.

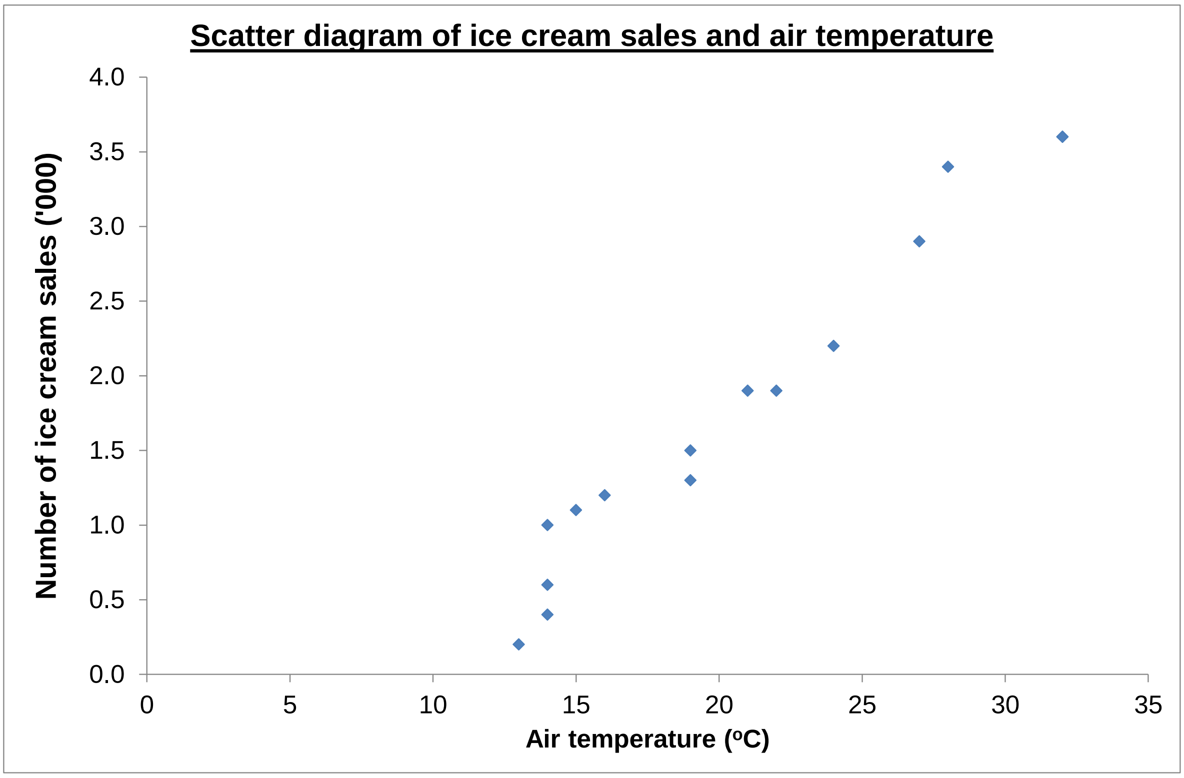
2. Mark scheme

(a) Award a maximum of 4 marks for appropriate commentary on the use of scatter diagrams, awarding: up to 2 marks for acknowledging their use in examining potential relationships between data; and up to a further 2 marks for any additional relevant point (e.g. reference to continuous data or use of examples).

Scatter diagrams are used as a visual aid to examine potential relationships between two sets of (quantitative) variables, rather than simply providing a visual summary of one (or more) set of variables. Therefore, scatter diagrams would be the most appropriate form of representing data graphically when you want to examine the relationship between two continuous variables (such as the relationship that may exist between sales revenue and the level of advertising expenditure).

(b) Award a maximum of 9 marks for an accurate fully labelled scatter diagram, awarding: 1 mark for a title; 1 mark for plotting the sales on the *y*-axis; up to 2 marks for the inclusion of labelled *x*- and *y*-axes, with the units (i.e. £'000) specified; up to 2 marks for a scaled and well-drawn scatter diagram; and up to 3 marks for accurately plotting the scatter points.
Award a maximum of 4 marks for appropriate commentary on the possible relationship, awarding: up to 2 marks for acknowledging the positive correlation; and up to a further 2 marks for any additional relevant point.

(i)



(ii) The scatter diagram suggests that there is a strong positive association (i.e. correlation) between air temperature and ice cream sales; as the air temperature rises, the number of ice cream sales increases. However, ice cream sales seem to drop out as temperature falls below 15ºC.

(c) Award 3 marks for the correct calculation. Award up to 2 method marks where the final answer is incorrect but there is evidence that the correct method has been used. If the volume of sales in part (i) is not presented in '000 units, then award a maximum of 2 marks and up to 1 method mark where the final answer is incorrect.

(i) $y = (0.1613 \times 25) - 1.5019$ $\quad y = 4.0325 - 1.5019 = 2.5306$ $\quad = 2{,}531$ sales

(ii) $3 = 0.1613x - 1.5019$ $\quad (3 + 1.5019) \div 0.1613 = x$ $\quad = 27.9^{\circ}C$

3. Recommendations

Students and tutors are advised to spend a little more time applying the general rules and principles of graphical construction as well as on the interpretation of scatter diagrams.

Examiner's tips

Always apply the general rules and principles of graphical construction: appropriate scaling; inclusion of axes labels (with units) and a title; accuracy of presentation; and a legend if necessary.

Question 5

(a) **List two examples of qualitative data and two examples of quantitative data.** **(4 marks)**

(b) **Classify each of the following sales data as either continuous or discrete:**

(i) **Total number of products sold** **(1 mark)**
(ii) **Weight of each product sold** **(1 mark)**
(iii) **Time taken to sell 100 products** **(1 mark)**
(iv) **Number of products sold by each employee** **(1 mark)**
(v) **Age of each employee working in the sales team** **(1 mark)**
(vi) **Distance that each product sold has to be transported** **(1 mark)**

(c) **The following data represents the weight in kilograms (kgs) of 18 products produced by a manufacturer on a given day:**

1.4	**3.1**	**3.8**	**5.8**	**4.9**	**4.4**
3.1	**5.2**	**3.1**	**0.7**	**5.6**	**4.7**
4.6	**3.7**	**2.2**	**5.3**	**4.5**	**2.2**

(i) **Draw a fully labelled stem and leaf diagram of product weights.** **(6 marks)**
(ii) **Using your stem and leaf diagram, determine the:**
- **modal product weight** **(2 marks)**
- **median product weight** **(2 marks)**
- **range of the product weights** **(2 marks)**

(iii) **Comment on the distribution of the product weight data.** **(3 marks)**

1. Comments on learners' performance

This question required students to distinguish between different types of data and to determine and interpret summary statistics.

Part (a) required students to list examples of quantitative and qualitative data. In the main this part was well attempted with relevant examples of the two terms presented. Similarly, students generally performed well classifying data as either 'continuous' or 'discrete' in part (b).

In part (c) some students did not attain full marks for their stem and leaf diagram because they failed to include a title and key. Although most students were able to use their stem and leaf diagram to interpret summary statistics, few students provided full comments on the distribution of the data.

2. Mark scheme

(a) Award 1 mark for each correct example, up to a maximum of 2 marks for each of the data types.
An example of quantitative data could include the number of cars produced by a car manufacturing plant each day. An example of qualitative data could include the different colours of cars produced by a car manufacturing plant each day.

(b) Award 1 mark for each correct classification of the types of data
(i) Discrete
(ii) Continuous
(iii) Continuous
(iv) Discrete
(v) Continuous
(vi) Continuous

(c)

(i) Award a maximum of 6 marks for an accurate fully labelled stem and leaf diagram, awarding: 1 mark for a title; 1 mark for the inclusion of a key; and up to 4 marks for a well-drawn and accurately plotted diagram.

Stem and leaf diagram of product weights (kgs)

0	7
1	4
2	2, 2
3	1, 1, 1, 7, 8
4	4, 5, 6, 7, 9
5	2, 3, 6, 8

5 | 8 represents 5.8 kgs

(ii) Award full marks for the correct calculation of the modal weight, median weight and range of weights. Award 1 method mark each where the final answer is incorrect but there is evidence that the correct method has been used.

Modal parcel weight	= 3.1 kgs
Median product weight is (3.8 + 4.4) ÷ 2	= 4.1 kgs
Range of product weights is 5.8 – 0.7	= 5.1 kgs

(iii) Award 1 mark for each appropriate comment made on the distribution of the product weight data, up to a maximum of 3 marks.

The distribution of product weights is negatively skewed; it has a long tail of lower product weights (i.e. to the left of the distribution)

3. Recommendations

Students and tutors are advised to spend a little more time applying the general rules and principles of graphical construction as well as on the interpretation of stem and leaf diagrams.

Examiner's tips

Ensure that you understand the difference between a negatively skewed distribution and a positively skewed distribution.

Question 6

(a) The following data show the number of employees working at ten grocery shops:

38 41 25 35 25 20 34 21 29 12

(i) **Calculate the mode, median and mean number of employees.** **(6 marks)**

(ii) **Calculate the range and standard deviation of the employee data.** **(7 marks)**

(iii) **Based on the measures of location and measures of dispersion calculated in (i) and (ii), comment on the distribution of the employee data.** **(3 marks)**

(b) The expenditure of 100 customers shopping at a food store on a given day is shown in the following table:

Expenditure (£)	Frequency
0 to less than 10	10
10 to less than 20	14
20 to less than 30	28
30 to less than 40	32
40 to less than 50	11
50 to less than 60	5

Using this data, calculate the:

(i) **Mean expenditure on food** **(3 marks)**

(ii) **Standard deviation of the food expenditure data** **(6 marks)**

1. Comments on learners' performance

This question concerned the application of statistical methods. Both parts of the question required students to calculate the measures of location and dispersion of a dataset, with part (a) concerning the application of statistical methods to an ungrouped dataset and part (b) concerning the application of statistical methods to a grouped dataset. Consistent with previous years, students generally performed less well calculating the summary statistics of a grouped dataset; common errors included not correctly computing the mid-values for each class interval, using *n* (i.e. the number of class intervals) as the value for $\sum f$, and a general lack of understanding of how to calculate $\sum fx$.

2. Mark scheme

(a) Award full marks for correctly calculating each measure of location and dispersion. Award method marks (1 mark for the calculation of the median and range, up to 2 marks for the mean and up to 3 marks for the standard deviation) where the final answer is incorrect but there is evidence that the correct method has been used. Do not penalise students for using an incorrect mean value in calculating the standard deviations in part a(ii), where the student has already lost marks for incorrectly calculating the mean in parts (i). In part a(iii), award 1 mark for each valid comment made, up to a maximum of 3 marks.

(i) Mode = 25 employees

Median $= \dfrac{29+25}{2}$ = 27 employees

Mean $= \dfrac{280}{10}$ = 28 employees

(ii) Range = 41 – 12 = 29 employees

Standard deviation $= \sqrt{\dfrac{8582}{10} - \left(\dfrac{280}{10}\right)^2}$ = 8.61 employees

(iii) The average number of employees working in the ten shops is 25, 27 or 28, depending on the measure used. Irrespective of the measure used, there is a high degree of variability in the number of employees, ranging from 12 to 41. As the mean is greater than the median and mode, this indicates that the distribution is not symmetrical but positively skewed.

(b) Award full marks for correctly calculating each measure of location and dispersion. Award method marks (up to 2 marks for the mean and up to 4 marks for the standard deviation) where the final answer is incorrect but there is evidence that the correct method has been used. Do not penalise students for using an incorrect mean value in calculating the standard deviations in part and b(ii), where the student has already lost marks for incorrectly calculating the mean in parts (i).

Expenditure (£)	Class mid-point (x)	Frequency (f)	fx	x^2	fx^2
0 to less than 10	5	10	50	25	250
10 to less than 20	15	14	210	225	3,150
20 to less than 30	25	28	700	625	17,500
30 to less than 40	35	32	1,120	1,225	39,200
40 to less than 50	45	11	495	2,025	22,275
50 to less than 60	55	5	275	3,025	15,125
		100	2,850		97,500

(i) Mean $= \frac{2,850}{100}$ = £28.50 **(3 marks MM up to 2 marks)**

(ii) Standard deviation $= \sqrt{\frac{97,500}{100} - \left(\frac{2,850}{100}\right)^2}$ = £12.76 **(6 marks MM up to 3 marks)**

3. Recommendations

Students and tutors are advised to spend a little more time calculating the mean and standard deviation of grouped data, and in particular how to calculate $\sum fx$ and $\sum f(x-\bar{x})^2$ when calculating the mean and standard deviation.

Examiner's tips

Familiarise yourself with the formulae for calculating the mean and standard deviation of grouped data, which is contained at the end of the examination paper.

Question 7

(a) A box contains 20 pieces of fruit, of which 15 are apples and 5 are oranges. A piece of fruit is randomly selected from the box. This piece of fruit is not put back in the box. A second piece of fruit is then randomly selected from the box of the remaining 19 pieces of fruit. Using this information:

> **(i) Draw a tree diagram to show the number of possible outcomes and their associated probabilities.**
>
> **(8 marks)**
>
> **(ii) Comment on whether all the possible outcomes are equally likely or not equally likely.**
>
> **(3 marks)**

(b) A production line manager carried out a random inspection of 300 products produced by two employees; 150 were produced by employee A and 150 were produced by employee B. The manager found that 30 of the products produced were of poor quality, with the remainder being of acceptable quality. Of the 150 products that were produced by employee A, 140 were found to be of acceptable quality. In contrast, 20 were found to be of poor quality out of the 150 products that were produced by employee B.

Using a contingency table or otherwise, calculate the probability that a product selected at random from these 300 products:

> **(i) Is of acceptable quality (3 marks)**
> **(ii) Was produced by employee A (3 marks)**
> **(iii) Was produced by employee A and is of poor quality (4 marks)**
> **(iv) Was produced by employee B, given that it is of acceptable quality (4 marks)**

1. Comments on learners' performance

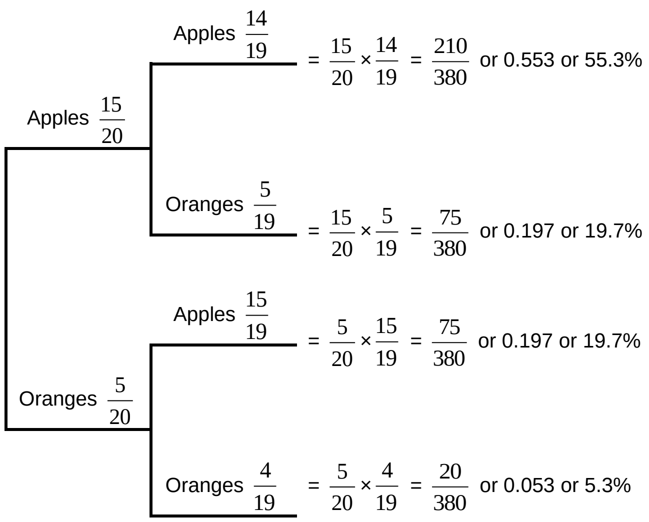
This question concerned the application of the laws of probability. Part (a) required students to draw a tree diagram. Most students were able to accurately conceptualise how to model the company's decision problem from the given set of possible outcomes and their probabilities using decision tree analysis.

Part (b) required the construction of a decision tree. Compared to previous examination sessions, the majority of students showed a good understanding of how to accurately construct the required contingency table for use in calculating probabilities. In contrast to previous examination sessions, a greater number of students showed difficulty determining probabilities than is normally the case. Students tended to show more difficulty calculating probabilities involving non-mutually exclusive and conditional events.

2. Mark scheme

(a) Award 8 marks for a correctly drawn tree diagram in part (i) which shows all four possible outcomes and their associated probabilities. Award up to 4 marks for the accurate construction of the decision tree showing the four possible outcomes and up to 4 marks for accurately calculating the probabilities associated with each outcome. In part (i), award 1 mark for each relevant comment up to a maximum of 3 marks.

(i)



(ii) Given that tree diagram clearly shows that the probabilities associated with each possible outcome is different, we can conclude that the possible outcomes are 'not equally likely'.

(b) Award full marks for correctly calculating each probability. Award up to 2 method marks where the final answer is incorrect but there is evidence that the correct method has been used.

	Employee A	Employee B	Total
Products of acceptable quality	140	130	270
Products of poor quality	10	20	30
Total	150	150	300

(i) P(acceptable quality) $= \frac{270}{300}$ $= \frac{9}{10}$ or 0.9 or 90%

(ii) P(employee A) $= \frac{150}{300}$ $= \frac{1}{2}$ or 0.5 or 50%

(iii) P(employee A & poor quality) $= \frac{10}{300}$ $= \frac{1}{30}$ or 0.03 or 3%

(iv) P(employee B | acceptable quality) $= \frac{130}{270}$ $= \frac{13}{27}$ or 0.48 or 48%

3. Recommendations

Students and tutors are advised to spend a little more time calculating probabilities involving non-mutually exclusive and conditional events.

Examiner's tips

Familiarise yourself with the formulae for calculating probabilities involving non-mutually exclusive and conditional events.

Question 8

(a) A company plans to sell ice cream at an outdoor event. If the weather is fine, then the company expects to make a profit of £6,000. However, if it rains then the event will be cancelled and the company will make a loss of £14,000. The weather forecast for the day of event is 15% possibility of rain.

Using this information, calculate the expected monetary value (EMV) in terms of the profit made by the sale of ice cream.

(5 marks)

(b) The daily demand (kgs) for bananas in a town is normally distributed with a mean of 1,000 kgs and a standard deviation of 100 kgs. Calculate the probability that the demand for bananas on a given day is:

- **(i) Less than 875 kgs** **(4 marks)**
- **(ii) More than 1,225 kgs** **(4 marks)**
- **(iii) Between 1,000 kgs and 1,225 kgs** **(4 marks)**
- **(iv) Between 1,100 kgs and 1,225 kgs** **(4 marks)**
- **(v) Between 900 kgs and 1,225 kgs** **(4 marks)**

1. Comments on learners' performance

This question concerned the application of the laws of probability. Part (a) required the calculation of the expected monetary value. In contrast to previous examination sessions, relatively few students were able to calculate the expected value.

Part (b) required students to determine probabilities using the normal distribution making use of tables. Students generally answered this part of the question very well. However, as in previous examination sessions students generally performed better when calculating probabilities to one side of the normal distribution (parts (i) and (ii)) compared to the calculation of a probability within a specific range (parts (iii), (iv) and (v)). Many students that did not correctly calculate the probability within the ranges specified showed some confusion of where within the normal distribution their calculated probabilities lay relative to the range specified.

2. Mark scheme

(a) Award a total of 5 marks for the correct expected monetary value. Award up to 3 marks where the final answer is incorrect but there is evidence that the correct method has been applied.

EMV = (6,000 x 0.85) + (−14,000 x 0.15) = £3,000

(b) Award 4 marks for correctly calculating each probability. Award up to 2 marks where the calculated probability is incorrect but there is evidence that the correct method has been used.

(i) $P(X < 875) = P(Z < \frac{875-1000}{100}) = P(Z < -1.25) = 0.1056$ or 10.56%

(ii) $P(X > 1{,}225) = P(Z > \frac{1225-1000}{100}) = P(Z > 2.25) = 0.01222$ or 1.222%

(iii) $P(1{,}000 < X < 1{,}225)$

$= P(\frac{1000-1000}{100} < Z < \frac{1225-1000}{100})$

$= P(0.00 < Z < 2.25)$

$= 0.5000 - 0.01222 = 0.48778$ or 48.778%

(iv) $P(1{,}100 < X < 1{,}225)$

$= P(\frac{1100-1000}{100} < Z < \frac{1225-1000}{100})$

$= P(1.00 < Z < 2.25)$

$= 0.1587 - 0.01222 \qquad = 0.14648$ or 14.648%

(v) $P(900 < X < 1{,}225)$

$= P\left(\frac{900-1000}{100} < Z < \frac{1225-1000}{100}\right)$

$= P(-1.00 < Z < 2.25)$

$= 1-(0.1587 + 0.01222) \quad = 0.82908$ or 82.908%

3. Recommendations

Students and tutors are advised to spend a little more time conceptualising business problems when calculating expected monetary values and determining probabilities using the normal distribution, particularly when calculating probabilities to one side of the normal distribution.

Examiner's tips

Remember to sum the expected values of each outcome when calculating the expected value of multiple outcomes.