Introduction to Quantitative Methods

NQF

Subject Examiner's Report

Unit Title:	Introduction to Quantitative Methods
Unit Code:	1.4 IQM
NQF Level:	3
Session:	December 2016

Comments on Learning Outcome and Assessment Criteria Performance

Learning Outcomes & Assessment Criteria	Comments
1 Demonstrate the rules of numeracy. 1.1 Apply the four rules to whole numbers, fractions and decimals 1.2 Express numbers in standard form 1.3 Multiply and divide negative numbers	Overall students performed very well meeting this Learning Outcome (LO). Students generally performed well meeting all the Assessment Criteria (AC).
2: Apply calculations 2.1 Compare numbers using ratios, proportions and percentages 2.3 Determine values for simple and compound interest, and for depreciation of an asset using the straight line method and the reducing balance method 2.5 Make calculations using a scientific calculator including roots and powers; logarithms and exponential values 2.7 Interpret, transpose and evaluate formulae	Overall students performed relatively well meeting this Learning Outcome (LO). Students generally performed better meeting Assessment Criterion (AC) 2.1 and 2.5, than they did with AC 2.3 and 2.7 which focused on the more on the application and interpretation of quantitative methods to business situations.
3: Use algebraic methods 3.1 Solve linear and simultaneous equations 3.2 Solve quadratic equations using factorisation and formulae 3.3 Solve equations using roots or logarithms 3.5 Determine the gradient and intercepts on the x or y axes for a straight line	Overall performance meeting this Learning Outcome (LO) was relatively weak. Students generally performed better meeting Assessment Criterion (AC) 3.1, 3.2 and 3.5, than they did with AC 3.3.
4: Construct and use: graphs, charts and diagrams 4.2 Plot graphs, applying general rules and principles of graphical construction including axes, choice of scale and zero 4.3 Plot and interpret mathematical graphs for simple linear, quadratic, exponential and logarithmic equations 4.4 Identify points of importance on graphs eg maximum, minimum and where they cut co-ordinate axes	Overall performance meeting this Learning Outcome (LO) was weak.
5: Apply statistical methods 5.1 Distinguish between quantitative and qualitative data 5.2 Distinguish between continuous and discrete random variables	Overall students performed relatively well meeting this Learning Outcome (LO). Students generally performed well meeting both Assessment Criterion (AC) 5.1 and 5.2.
6: Apply the laws of probability 6.1 Recognise outcomes which are equally likely, not equally likely or subjective 6.2 Use appropriate formulae to determine probabilities for complementary, mutually exclusive, independent and conditional events 6.3 Determine probabilities using sample space, two way table or tree diagram 6.4 Determine the expected value of a variable 6.5 Determine probabilities using the normal distribution making use of tables 6.6 Represent normal probabilities as areas under the standard normal curve	Overall performance meeting this Learning Outcome (LO) was weak. Students generally performed better meeting Assessment Criterion (AC) 6.3, 6.4, 6.5 and 6.6, than they did with AC 6.1 and 6.2.

Question 1

(a) Without the use of a calculator, find the value of the following, showing all steps in your calculations:

(i) $\frac{33}{8} \div \frac{3}{4}$

(ii) $\frac{(-4)\times(-5)}{8-(-2)}$

(iii) $\left(\frac{1}{2}\right)^2 \times \sqrt{\frac{9}{4}}$ (9 marks)

(b) Use a calculator to find the value, correct to 2 decimal places, of:

(i) $(-0.5) \times e^{1.39}$

(ii) $\ln(56) \div (-4)$ (6 marks)

(c) The sales revenue of a British food retailing company in the financial year 2013/14 was £7,755,941.56. Express this sales revenue:

(i) To the nearest £

(ii) Correct to 4 significant figures

(iii) In standard form $A \times 10^n$ (where $1 \leq A < 10$ and n is an integer) (6 marks)

(d) The world price of crude oil has risen by approximately 140% over the last five years. Express this percentage rise as a:

(i) Decimal

(ii) Fraction in its simplest form (4 marks)

Total 25 Marks

Learning Outcome 1: Demonstrate the rules of numeracy

1.1 Apply the four rules to whole numbers, fractions and decimals

1.2 Express numbers in standard form

1.3 Multiply and divide negative numbers

Learning Outcome 2: Apply calculations

2.1 Compare numbers using ratios, proportions and percentages

2.5 Make calculations using a scientific calculator including roots and powers; logarithms and exponential values

2.7 Interpret, transpose and evaluate formulae

1. Mark scheme

(a) To gain full marks, all worked steps must be shown. The use of a calculator is not permitted. Award 1 mark for each correct answer and up to 3 marks for each correct answer where full workings are shown. Where the final answer is incorrect but the correct method has been used, award up to 2 marks for each answer.

(b) Award 3 marks for each correct answer and 2 marks for each answer where the final answer is correct but not expressed correctly to 2 decimal places. Where the final answer is incorrect but the correct method has been used, award up to 2 marks for each answer.

(c) Award 2 marks for each correct answer and 1 mark for each answer where the final answer is incorrect but there is evidence that the correct method has been used.

(d) Award 2 marks for each correct answer and 1 mark for each answer where the final answer is incorrect but there is evidence that the correct method has been used. Award a maximum of 1 mark if the fraction in part (ii) is not expressed in its simplest form.

2. Model Answer

(a)

(i) $\dfrac{33}{8} \div \dfrac{3}{4}$ $= \dfrac{33}{8} \times \dfrac{4}{3}$ $= \dfrac{132}{24}$ = 5.5

(ii) $\dfrac{(-4)\times(-5)}{8-(-2)}$ $= \dfrac{20}{10}$ = 2

(iii) $\left(\dfrac{1}{2}\right)^2 \times \sqrt{\dfrac{9}{4}}$ $=$ $\dfrac{1}{4} \times \dfrac{3}{2}$ $= \dfrac{3}{8}$

(b)

(i) (–0.5) × (4.01) = –2.01

(ii) 4.03 ÷ (–4) = –1.01

(c)

(i) 7,755,942

(ii) 7,756,000

(iii) 7.75594156×10^6

(d)

(i) 1.4

(ii) $1\frac{2}{5}$

3. Comments on learners' performance

As in previous examination sessions, this was the most popular question on the paper, with most students scoring reasonably good marks.

Part (a) required the application of the four rules of numeracy to fractions, including the use of square roots. While most students performed very well in part (i), a number of students did not understand how to multiply and divide fractions. In part (ii) students also performed well, although a number of students failed to understand the implication of the brackets and the two minus signs with many students incorrectly multiplying instead of adding the '8' and '2' on the denominator. Part (iii) was answered less well, with many students showing difficulty taking the square root of the fraction and carrying out the arithmetic operations in the correct sequence. Common to parts (i) to (iii), a minority of students failed to show all relevant steps in their calculations and/or failed to express each answer as a fraction in its simplest form as stipulated in the question.

Part (b) required the use of a scientific calculator to make calculations involving logarithms and exponentials. This part was generally well answered, with relatively few students exhibiting any significant problems; students generally found the calculation involving the logarithm easier than that involving the exponential. Where students typically lost marks, this was generally because their answers were not expressed correct to 2 decimal places as requested.

Part (c) required data to be expressed in different formats. This generally did not pose too many problems, although expressing data using significant figures tended to be weaker.

Part (d) required a percentage to be expressed as a decimal and a fraction in its simplest form. In general, students showed few problems, although a common mistake made by many students was to divide/express the percentage over a base of '1000'.

4. Recommendations

Students and tutors are advised to spend a little more time applying the four rules of numeracy without the use of a calculator, focusing in particular on the multiplication and division of fractions, the subtraction of negative numbers and the importance of correctly sequencing arithmetic operations.

Examiner's tips

Do not forget to show all relevant steps in your calculations and always express fractions in their simplest form.

Question 2

(a) A steel manufacturing company purchases new equipment costing £150,000. Calculate the value of the equipment after five years, if it is depreciated by:

(i) £18,000 per year, using the straight line method.

(5 marks)

(ii) 18% per year, using the reducing balance method.

(5 marks)

(b) A sum of £50,000 is to be invested over an 8 year period in two different banks that offer different interest rates and terms. Half of the amount is to be invested in Bank A, which pays simple interest at 6.0% per annum. The remainder is to be invested in Bank B, which pays compound interest at 5.9% per annum.

(i) Calculate the total interest that would be received from each bank after the 8 year investment period. (Give your answers to the nearest £).

(10 marks)

(ii) Calculate the annual rate of compound interest that would be necessary in order for £25,000 to grow to £40,000 by the end of 8 years. (Give your answer correct to 1 decimal place).

(5 marks)

Total 25 marks

Learning Outcome 2: Apply calculations

2.3 Determine values for simple and compound interest, and for depreciation of an asset using the straight line method and the reducing balance method

2.7 Interpret, transpose and evaluate formulae

1. Mark scheme

(a) Award 5 marks for each correct answer to parts (i) and (ii) and up to 3 marks where the final answer is incorrect but there is evidence that the correct method has been used.

(b) For part (i) award 5 marks for each correct answer and up to 3 marks where the final answer is incorrect but there is evidence that the correct method has been used. If the invested sum has not been subtracted in calculating compound interest, award a maximum of 4 marks.

For part (ii), award 5 marks for the correct interest rate and up to 3 marks where the final answer is incorrect but there is evidence that the correct method has been used. If the interest rate is not given to 1 decimal place, award a maximum of 4 marks.

2. Model Answer

(a)

(i) Initial cost: £150,000

Depreciation:	£18,000 × 5	= £90,000
Value of machine after four years:	£150,000 – £90,000	= £60,000

(ii) The value at the end of each year is 82% of the value at the beginning of the year:

Value at the end of year 1:	£150,000.00 × 82%	= £123,000.00
Value at the end of year 2:	£123,000.00 × 82%	= £100,860.00
Value at the end of year 3:	£100,860.00 × 82%	= £82,705.20
Value at the end of year 4:	£82,705.20 × 82%	= £67,818.26
Value at the end of year 5:	£67,818.26 × 82%	= £55,610.98

Or by formula, the value of the machine after five years:

$= 150{,}000 \times (1\text{-}0.18)^5$

$= 150{,}000 \times 0.80^5$ = £55,610.98

(b)

(i) Bank A: $25{,}000 \times \left(8 \times \frac{6}{100}\right)$ = £12,000

Bank B: $25{,}000 \times \left(1 + \frac{5.9}{100}\right)^{8}$ - 25,000 = £14,546

(ii) $25{,}000 \times \left(1 + \frac{i}{100}\right)^{8} = 40{,}000 \quad \therefore \left(1 + \frac{i}{100}\right)^{8} = \frac{40{,}000}{25{,}000} \quad \therefore \left(1 + \frac{i}{100}\right)^{8} = 1.6$

$\therefore 1 + \frac{i}{100} = 1.6^{\frac{1}{8}} \quad \therefore 1 + \frac{i}{100} = 1.060511 \quad \therefore i = (1.060511 - 1) \times 100 \quad \therefore i = 6.1\%$

3. Comments on learners' performance

This was a relatively popular question. It concerned the application of quantitative methods to business situations, notably calculations involving depreciation rates and interest rates.

Part (a) concerned the calculation of the future value of an asset using both the straight line and reducing balance methods. Compared to previous examinations, many students had difficulty answering this question, with a significant number of students unable to correctly apply the straight line method. Students also lost marks for treating the calculated value for depreciation as the future value of the asset.

Part (b) required students to calculate simple and compound interest using two different interest rates. As in previous examinations, a very common error made by students in calculating the compound interest that would be received was to present the final value of the investment with interest, rather than just the interest received. In addition, many students that did answer this question correctly lost marks because they did not present the final answer to the nearest £ as requested.

This part also required students to rearrange the formula for calculating compound interest to solve for the annual interest rate, given that the term of the investment, the original principal and the accrued amount were known. A significant proportion of students were unable to re-arrange the formula to calculate the compound interest rate.

4. Recommendations

Students and tutors are advised to spend a little more time learning how to use different formulae for different business applications, notably the use of the straight line method to calculate the depreciated value of an asset and how to re-arrange the formula to calculate for the compound interest rate.

Examiner's tips

Try to visualise the mathematical problem being examined. You should be able to tell if your answer is likely to be correct even without knowing the answer.

Question 3

(a) The demand for a product can be represented by the equation $P = 900 - 0.3Q$, where P is the market price of the product (£ per unit) and Q is the quantity (units) demanded in a given period.

(i) From the equation, determine the co-ordinates at which the demand curve would intersect the y-axis if plotted on a graph. (2 marks)

(ii) From the equation, determine the gradient of this demand curve. (2 marks)

(iii) Using the equation, calculate the market price of the product if 500 units were demanded. (3 marks)

(iv) Using the equation, calculate the number of units demanded if the market price of the product was £600 per unit. (3 marks)

(b) Solve the following equations:

(i) $6x - 4 = 3x + 11$ (2 marks)

(ii) $\frac{x}{6} + \frac{x}{3} = \frac{12}{6}$ (4 marks)

(iii) $x^2 + 3x + 2 = 0$, using factorisation (4 marks)

(c) Simplify the following logarithm equation to a single log term: $\log(x - 24) + \log(x)$ (5 marks)

Total 25 Marks

Learning Outcome 3: Use algebraic methods

3.1 Solve linear and simultaneous equations
3.2 Solve quadratic equations using factorisation and formulae
3.3 Solve equations using roots or logarithms
3.5 Determine the gradient and intercepts on the x or y axes for a straight line

1. Mark scheme

(a) Award full marks for the correct answer. Award up to 1 method mark each for parts (i) and (ii) and up to 2 method marks each for parts (iii) and (iv), where the final answer is incorrect but there is evidence that the correct method has been used.

(b) Award full marks for the correct answer. Award up to 1 method mark for part (i) and up to 2 method marks each for parts (ii) and (iii), where the final answer is incorrect but there is evidence that the correct method has been used. If the quadratic formula has been used instead of factorisation in part (iii), then award a maximum of 2 marks for the correct answer.

(c) Award 5 marks for a correct answer and up to 3 method marks where the final answer is incorrect but there is evidence that the correct method has been used. Award 4 marks where the final answer is correct but given without the brackets - i.e. $\log x^2 - 24x$.

2. Model Answer

(a)

(i) (0, 900)

(ii) −0.3

(iii) $P = 900 - (0.3 \times 500)$ $\quad P = 900 - (150)$ $\quad P = £750$

(iv) $600 = 900 - 0.3Q$ $\quad 0.3Q = 900 - 600$ $\quad Q = \frac{900-600}{0.3}$ $\quad Q = 1{,}000$ units

(b)

(i) $6x - 4 = 3x + 11$ $\quad 6x - 3x = 11 + 4$ $\quad 3x = 15$ $\quad x = 5$

(ii) $\frac{x}{6} + \frac{x}{3} = \frac{12}{6}$ $\quad \frac{x}{6} + \frac{2x}{6} = \frac{12}{6}$ $\quad \frac{3x}{6} = \frac{12}{6}$

$x = \frac{12}{6} \times \frac{6}{3}$ $\quad x = \frac{72}{18}$ $\quad x = 4$

(iii) $x^2 + 3x + 2 = 0$, using factorisation gives: $\quad (x + 2)(x + 1) = 0$

$\therefore$	either:	$x + 2 = 0$	or	$x + 1 = 0$
so:		$x = -2$	or	$x = -1$

(c) Using the rule: $\log (p \times q) = \log p + \log q$

$$\log (x - 24) + \log (x) \qquad = \log (x (x - 24)) \qquad = \log (x^2 - 24x)$$

3. Comments on learners' performance

This question concerned the use of algebraic methods and logarithms. In general, this question was relatively well answered with a high number of students attaining good grades.

Part (a) required students to evaluate a demand equation and use it to solve business problems. Most students performed extremely well on this question.

Part (b) required students to solve three equations. Students performed very well solving the equation in part (i) and solving the quadratic equation using factorisation in part (iii); a few students used the quadratic formulae to solve this equation instead of factorisation as stated in the question. In contrast, they generally performed least well solving the equation where x was expressed as a fraction.

Part (c) required students to simplify a logarithm equation to a single log term. Compared to previous examinations, fewer students were able to simplify this logarithm correctly; often unaware of which logarithm rules to use. Nevertheless, a good number of students made a good attempt at applying the appropriate logarithm rules to simplify the equation to a single log term.

4. Recommendations

Students and tutors are advised to spend a little more time rearranging equations to solve for x, where x is expressed as a fraction, as well as simplifying logarithm expressions to a single log term, with a particular focus on choice and application of the appropriate logarithm rules.

Examiner's tips

If a question states that you should solve a quadratic equation using factorisation, then do not instead use the quadratic formula as this will likely mean that you will not be able to attain full marks.

Question 4

(a) The supply curve for a particular brand of paint can be represented by the equation $P = 10 + 2Q_S$, where P is the price of the product (£ per unit) and Q_S is the quantity supplied (units) in a given period. The demand curve for the same brand of paint can be represented by the equation $P = 50 - 0.5Q_D$, where Q_D is the quantity demanded (units) in a given period.

(i) Looking at the demand equation, determine the gradient of this product's demand curve.

(2 marks)

(ii) Looking at the supply equation, determine the co-ordinates at which the supply curve would intersect the y-axis.

(2 marks)

(iii) Use the demand and supply equations to plot a fully labelled line graph. Use the graph paper provided at the front of your answer book.

(9 marks)

(iv) Use your graph to determine the equilibrium price and quantity of brand of paint (i.e. the price and quantity at which the demand and supply equations intercept).

(2 marks)

(b) The annual cost of producing the brand of paint by a paint manufacturing company can be represented by the equation $C = 30{,}000 + 0.5x$, where C is the total cost (£) of producing the branded paint and x is the quantity of tins of paint produced.

(i) Looking at the cost equation, determine the value of fixed costs (i.e. those costs that remain constant irrespective of the quantity produced).

(2 marks)

(ii) Looking at the cost equation, determine the value of the variable costs of production per tin of paint produced.

(2 marks)

(iii) Use the cost equation to calculate the total cost of production if the quantity of paint produced is 100,000 tins.

(3 marks)

(iv) Use the cost equation to calculate the quantity (tins) of paint produced given that the total cost of production is £120,000.

(3 marks)

Total 25 Marks

Learning Outcome 3: *Use algebraic methods*

3.1 Solve linear and simultaneous equations

3.5 Determine the gradient and intercepts on the x or y axes for a straight line

Learning Outcome 4: Construct and use: graphs, charts and diagrams

4.2 Plot graphs, applying general rules and principles of graphical construction including axes, choice of scale and zero

4.3 Plot and interpret mathematical graphs for simple linear, quadratic, exponential and logarithmic equations

4.4 Identify points of importance on graphs eg maximum, minimum and where they cut co-ordinate axes

1. Mark scheme

(a) For Part (i), award 2 marks for the correct answer and 1 mark where the gradient is presented as a positive (i.e. 0.5). If the gradient of the supply equation is given instead, award 1 mark.
For Part (ii), award 2 marks for correctly determining both co-ordinates, and 1 mark where only one co-ordinate has been given. If both co-ordinates of the demand equation are given instead, award 1 mark.
For Part (iii), award up to 9 marks for a titled and fully labelled line graph. Specifically, award: up to 5 marks for a well drawn, accurate and scaled graph; 1 mark for the inclusion of a title; up to 2 marks for accurately labelled *x* and *y*-axes with axis units specified; and 1 mark for the inclusion of a legend or appropriate labelling of the supply/demand curves.
For Part (iv), award 2 marks for determining both the equilibrium price and quantity, and 1 mark where only the equilibrium price or the quantity has been given.

(b) For parts (i) and (ii), award 2 marks for each correct answer and 1 mark for each answer where the final answer is incorrect but there is evidence that the correct method has been used.

For parts (iii) and (iv), award 3 marks for each correct answer and up to 2 marks for each answer where the final answer is incorrect but there is evidence that the correct method has been used.

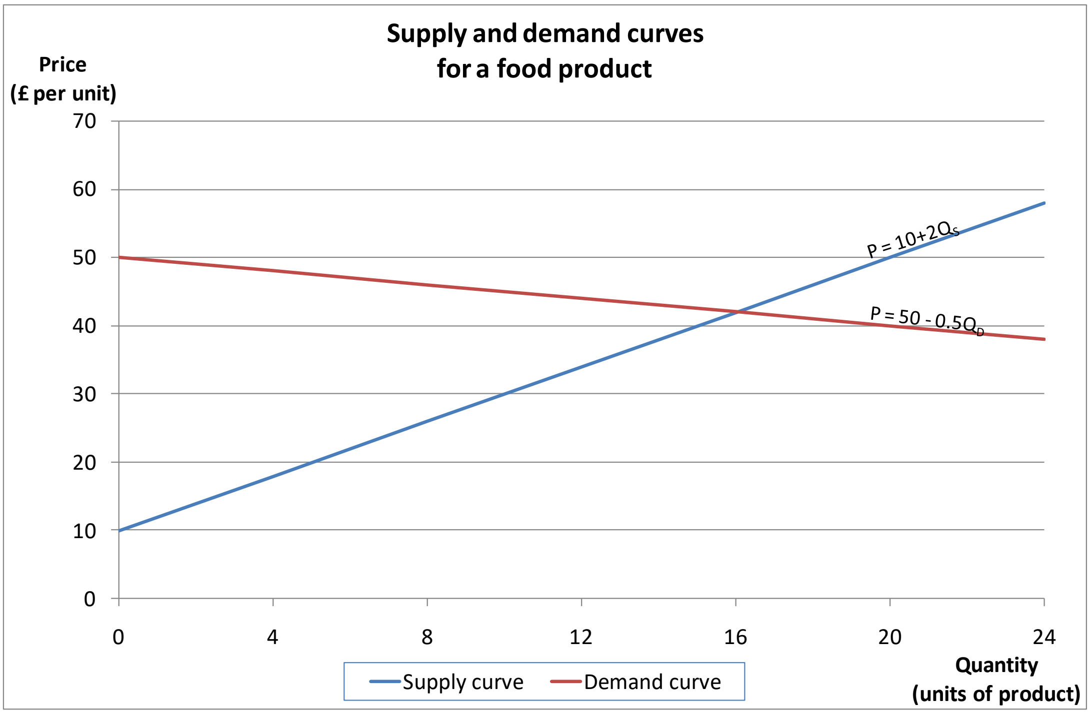
2. Model Answer

(a)

(i) Gradient is –0.5

(ii) Co-ordinates are (0,10).

(iii)



(iv) At equilibrium, P = £42 per unit and Q = 16 units.

(b)

(i)	Fixed costs	= £30,000
(ii)	Variable costs	= £0.50 per tin of paint
(iii)	$C = 30{,}000 + 0.5x$	$C = 30{,}000 + (0.5 \times 100{,}000)$ $C = 30{,}000 + 50{,}000$ $C = £80{,}000$
(iv)	$C = 30{,}000 + 0.5x$	$120{,}000 = 30{,}000 + 0.5x$ $120{,}000 - 30{,}000 = 0.5x$ $90{,}000 = 0.5x$ $90{,}000 \div 0.5 = x$ $x = 180{,}000$ tins of paint

3. Comments on learners' performance

This question concerned the conceptualisation of business problems through the use of algebraic methods and the construction and use of graphs.

Part (a) required students to interpret and plot linear equations. While generally students performed well answering parts (i) and (ii), a number of students showed confusion as to which part of each equation represented the intercept and which represented the gradient. Another common mistake when presenting the gradient was the failure to include the minus sign to demonstrate that the gradient was downward sloping. In general, students made a fair attempt at constructing and interpreting their graph. Consistent with previous examination sessions, students typically lost marks for failing to include a title, and axes labels and units.

The final part of the question required students to interpret points on their graphs. Common problems included: expressing the equilibrium quantity and price in part (i) in the correct units (notably Q=20,000 and not 20); determining and expressing the co-ordinates at which the supply curve intersects the y-axis in (ii) in the form (0,10); and forgetting to express the gradient of the demand curve in (iii) as a negative.

Part (b) required students to interpret and solve a linear cost equation. Students generally performed less well on this part of the question. Students generally showed confusion as to which part of the cost equation represented those costs that were fixed and those that were variable in parts (i) and (ii). Students also generally showed confusion solving (and rearranging) the equation for *C* and *x*.

4. Recommendations

Students and tutors are advised to spend a little more time interpreting, solving and plotting graphs for simple linear equations.

Examiner's tips

When rearranging formulae, pay particular attention to the implication of any minus signs.

Question 5

(a) Explain the difference between 'quantitative data' and 'qualitative data', using examples.

(4 marks)

(b) Classify the following operational data, used by a delivery company to monitor performance, as either continuous or discrete:

- (i) Average number of parcels delivered per hour
- (ii) Time taken to deliver ten parcels
- (iii) Total weight of parcels delivered each day
- (iv) Distance travelled during the day by each delivery driver

(4 marks)

(c) Given the quadratic equation $y = x^2 - 8x + 12$:

- (i) Construct a table and calculate the value of y for the following values of x: –1, 0, 1, 2, 3, 4, 5, 6, 7, 8.

(5 marks)

- (ii) Using your tabulated data in (i), plot a graph of $y = x^2 - 8x + 12$ for the values of x from $x = -1$ to $x = 8$. (Use the graph paper at the front of your answer book.)

(6 marks)

(d) Using the graph of $y = x^2 - 8x + 12$ plotted in (c), find the:

- (i) values of x when $y = 0$
- (ii) values of x and y when $y = x^2 - 8x + 12$ is at its minimum
- (iii) co-ordinates at which the function $y = x^2 - 8x + 12$ intersects the y-axis

(6 marks)

Total 25 Marks

Learning Outcome 4: Construct and use: graphs, charts and diagrams

4.2 Plot graphs, applying general rules and principles of graphical construction including axes, choice of scale and zero

4.3 Plot and interpret mathematical graphs for simple linear, quadratic, exponential and logarithmic equations

4.4 Identify points of importance on graphs eg maximum, minimum and where they cut co-ordinate axes

Learning Outcome 5: Apply statistical methods

5.1 Distinguish between quantitative and qualitative data

5.2 Distinguish between continuous and discrete random variables

1. Mark scheme

(a) Award 1 mark for each valid point made in differentiating between the two types of data, up to a maximum of 2 marks. Award 1 mark for each example of 'quantitative data' and 'qualitative data' given, up to a maximum of 2 marks.

(b) Award 1 mark for each correct answer.

(c) For part (i) award 5 marks for a correctly constructed table and up to 3 marks if the contingency table is not totally correct but there is evidence of some correct workings.
For part (ii), award 6 marks for an accurate fully labelled plot. Award 1 mark for the inclusion of a title and 1 mark for axes labels. Award up to 4 marks for an accurate, scaled and well-drawn plot of the quadratic equation.
(Accept OF marks for part (ii) where a student has not attained full marks in part (i) for incorrectly calculated y-values, but where these have been correctly plotted in part (ii)).

(d) Award 2 marks for each part (i) to (iii), with 1 mark for each correct value.

2. Model Answer

(a) Quantitative data is generally referred to as information about 'quantities', for which observations are measurable and numerical in nature. Such data is easy to analyse statistically. In contrast, qualitative data is generally referred to as information about 'qualities', for which observations can't actually be measured and are non-numeric in nature.
An example of quantitative data could include the number of cars produced by a car manufacturing plant each day. An example of qualitative data could include the different colours of cars produced by a car manufacturing plant each day.

(b)

- (i) Discrete
- (ii) Continuous

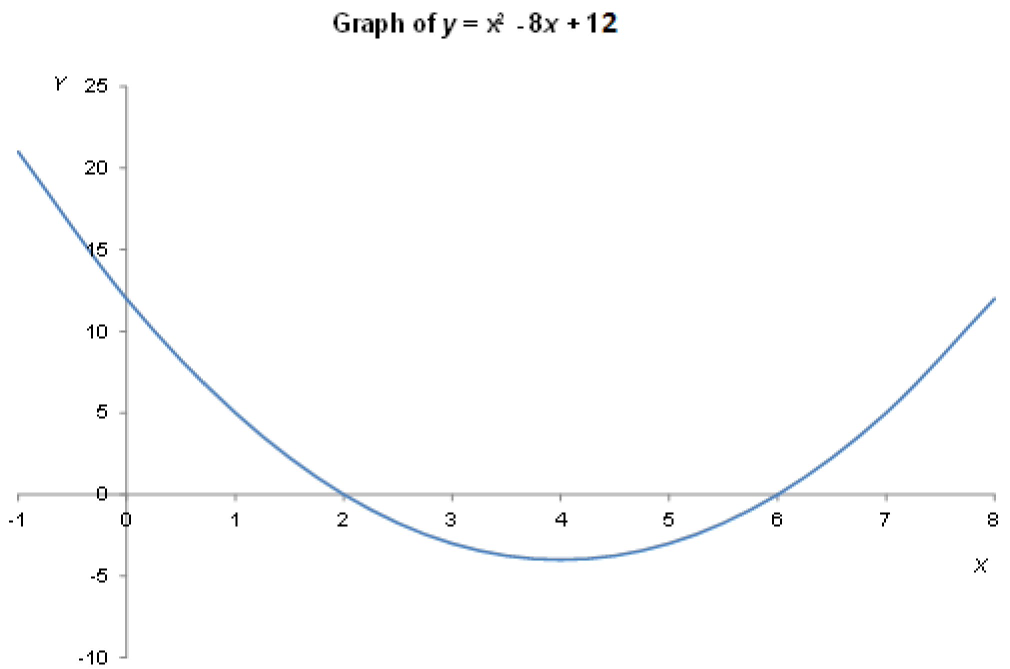
(iii) Continuous
(iv) Continuous

(c)

(i)

x	-1	0	1	2	3	4	5	6	7	8
x^2	1	0	1	4	9	16	25	36	49	64
$-8x$	8	0	-8	-16	-24	-32	-40	-48	-56	-64
12	12	12	12	12	12	12	12	12	12	12
y	21	12	5	0	-3	-4	-3	0	5	12

(ii)



(d)

(i) $x = 2$ and 6
(ii) $x = 4$ and $y = -4$
(iii) (0, 12)

3. Comments on learners' performance

This question required students to distinguish between different types of data and to plot and interpret mathematical graphs for a quadratic equation.

Part (a) required students to differentiate between quantitative and qualitative data, with examples. In the main this part was well attempted with good definitions and examples of the two terms presented.

Part (b) required students to classify data as either continuous or discrete. In the main this part was well attempted.

Part (c) required students to use a quadratic equation to calculate the value of y for a range of x values, and then to use this data to plot a fully labelled graph of their quadratic equation. Most students correctly calculated the value of y for the values of x = 0 to 8, although for the value of x = -1 this proved more difficult for a number of students. In the main, students made a good attempt at plotting the graph. Consistent with previous examination sessions, students typically lost marks for: not including a title; not labelling the x and y axes; and not drawing a 'smoothed' line through the data points.

Part (d) required students to identify certain points on their graph. Most students were able to easily identify the co-ordinates at which the quadratic function intersected the y-axis (part (iii)). However, fewer students were able to identify the values of x when y = 0 (part (i)) and the values of x and y when the function was at its minimum (part (ii));

particularly where students had drawn a horizontal line to connect the co-ordinates at the bottom of their plotted function, resulting in no single minimum set of co-ordinates.

4. Recommendations

Students and tutors are advised to spend a little more time constructing tables and using quadratic equations to calculate the value of *y*, as well as applying the general rules and principles of graphical construction and the interpretation of graphs.

Examiner's tips

When plotting graphs, ensure you accurately plot each data point.

Question 6

(a) Explain, using an example, what is meant by the term 'equally likely outcome'. (5 marks)

(b) An architect hopes to win two contracts in 2016, which are to be awarded independently of each other. The architect predicts that her chances of being awarded contract A is 30% and contract B is 15%. Contract A will generate an income of £40,000 and contract B will generate an income of £20,000.
Using this information, calculate the 'expected monetary value' of the revenue to the consulting company from both contracts A and B. (4 marks)

(c) A drinks company produces 5,000 bottles of apple juice per day, with each bottle containing 1 litre of juice. On a given day, 200 bottles were found to contain the wrong volume of juice, of which 50 bottles contained less than 1 litre.

(i) If one bottle is randomly selected from the daily production of 5,000 bottles, calculate:
- The probability that it will contain less than 1 litre of juice. (4 marks)
- The probability that it will contain more than 1 litre of juice. (4 marks)

(ii) If two bottles are randomly selected from the daily production of 5,000 bottles, calculate:
- The probability that both bottles will each contain exactly 1 litre of juice. (4 marks)
- The probability that both bottles will each contain less than 1 litre of juice. (4 marks)

Total 25 Marks

Learning Outcome 6: Apply the laws of probability

6.1 Recognise outcomes which are equally likely, not equally likely or subjective

6.2 Use appropriate formulae to determine probabilities for complementary, mutually exclusive, independent and conditional events

6.4 Determine the expected value of a variable

1. Mark scheme

(a) Award a maximum of 5 marks for an appropriate explanation and example of equally likely outcomes.

(b) Award a total of 4 marks for the correct expected monetary value. Award up to 2 marks where the final answer is incorrect but there is evidence that the correct method has been applied.

(c) Award full marks for correctly calculating the probabilities to each part, and method marks as shown below where the final answer is incorrect but there is evidence that the correct method has been used. Do not penalise students for using an incorrect probability calculated in (i) when calculating the probability in part (ii).

2. Model Answer

(a) An equally likely outcome is one in which any one outcome of an event is no more likely to occur than any other. For example, the possible outcomes of tossing a coin are either 'heads' or 'tails'. As long as the coin is unbiased, then we would not expect 'heads' to occur any more than 'tails' when tossing the coin. In practice, equally likely outcomes are not common.

(b) EMV = (40,000 x 0.3) + (20,000 x 0.15) = £15,000

(c)

(i) P (bottle < 1 litre) $= \frac{50}{5,000} = \frac{1}{100}$ or 0.01 or 1%

P (bottle > 1 litre) $= \frac{150}{5,000} = \frac{3}{100}$ or 0.03 or 3%

(ii) P (both bottles = 1 litre) $= \frac{4,800}{5,000} \times \frac{4,799}{4,999} = \frac{23,035,200}{24,995,000}$ or 0.92 or 92%

P (both bottles < 1 litre) $= \frac{50}{5,000} \times \frac{49}{4,999} = \frac{2,450}{24,995,000}$ or 0.000098 or 0.0098%

3. Comments on learners' performance

This question concerned the application of the laws of probability. As in previous examinations, this is one of the least popular questions on the exam paper.

Part (a) required students to explain what an 'equally likely outcome' is. Most students showed difficulty providing an appropriate explanation of the term, with few appropriate examples provided.

Part (b) required the calculation of the expected monetary value. In contrast to previous examination sessions, this question was answered well with most students demonstrating an excellent understanding of how to calculate expected values. Of those students that failed to calculate the expected value correctly, many showed an understanding of how to calculate expected values of a single outcome but failed to sum the expected sales values from each outcome as required.

Part (c) required students to calculate various probabilities. Students generally made a good attempt at determining the probabilities in part (i), although consistent with previous examinations students tended to show more difficulty calculating probabilities in part (ii) which required the use of the multiplication rule.

4. Recommendations

Students and tutors are advised to spend a little more time conceptualising business problems and applying the various laws of probability to determine probabilities and expected outcomes. In addition, students should gain a better understanding of the term 'equally likely outcome' in the context of probability theory.

Examiner's tips

Familiarise yourself with the formulae for calculating the different types of probabilities, which are presented at the back of the Examination Question Paper

Question 7

(a) Explain the difference between 'continuous' and 'discrete' variables, giving an example of each.

(6 marks)

(b) A box contains 20 balls, of which 15 are green balls and 5 are orange balls. A ball is selected at random from the box and its colour noted. This ball is not put back in the box. A second ball is then randomly selected from the box of the remaining 19 balls and its colour noted.

(i) Given this information, draw a tree diagram to show the number of possible outcomes and their associated probabilities. (8 marks)

(ii) Using your tree diagram or otherwise, determine the probability that the two balls selected are both:

- Green (1 mark)
- Orange (1 mark)
- Different colours (2 marks)

(c) The probability that a marketing campaign being run by a small food company will generate an additional 1,000 sales is 0.75 and the probability that it will generate an additional 10,000 sales is 0.25. Calculate the additional expected sales from carrying out this marketing campaign. (4 marks)

(d) For the following events, classify whether all the possible outcomes are 'equally likely' or 'not equally likely':

(i) Tossing a coin

(ii) Rolling a six-sided dice

(iii) Selecting students based on nationality (3 marks)

Total 25 Marks

Learning Outcome 5: Apply statistical methods

5.2 Distinguish between continuous and discrete random variables

Learning Outcome 6: Apply the laws of probability

6.1 Recognise outcomes which are equally likely, not equally likely or subjective

6.3 Determine probabilities using sample space, two way table or tree diagram

6.4 Determine the expected value of a variable

1. Mark scheme

(a) Award 1 mark for each valid point made in differentiating between the two types of variables, up to a maximum of 4 marks. Award 1 mark for each example of a 'continuous' and 'discrete' variable given, up to a maximum of 2 marks.

(b) For part (i) award 8 marks for a correctly drawn tree diagram which shows all four possible outcomes and their associated probabilities. Award up to 4 marks for the accurate construction of the decision tree showing the four possible outcomes and up to 4 marks for accurately calculating the probabilities associated with each outcome. For part (ii) award a total of 4 marks for correctly deriving the three outcome probabilities using the tree diagram (or otherwise). Do not penalise students for any errors carried forward from part (i) where the student has already lost marks in part (i).

(c) Award 4 marks for the correct answer and up to 2 marks where the final answer is incorrect but there is evidence that the correct method has been used.

(d) Award 1 mark for each correct answer.

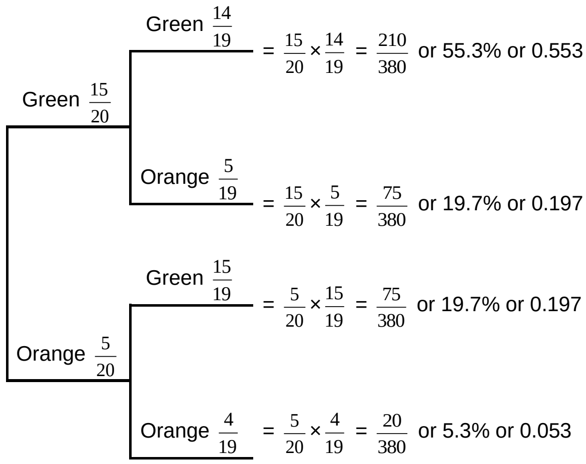
2. Model Answer

(a) Variables may be either continuous or discrete. A continuous variable may take any value between two stated limits (which may possibly be minus and plus infinity). For example, height is a continuous variable because a person's height may (with appropriately accurate equipment) be measured to any minute fraction of a millimetre.

A discrete variable, however, can take only certain values occurring at intervals between stated limits. For most (but not all) discrete variables, these interval values are the set of integers (whole numbers). For example, if the variable is the number of children per family, then the only possible values are 0, 1, 2, etc. because it is impossible to have other than a whole number of children. However, in Britain, shoe sizes are stated in half-units, and so here we have an example of a discrete variable which can take the values 1, 1½, 2, 2½, etc.

(b)

(i)



(ii) P(both green) $= \left(\frac{15}{20} \times \frac{14}{19}\right) = \frac{210}{380}$ or 55.3% or 0.553

P(both orange) $= \left(\frac{5}{20} \times \frac{4}{19}\right) = \frac{20}{380}$ or 5.3% or 0.053

P(both different) $= \left(\frac{15}{20} \times \frac{5}{19}\right) + \left(\frac{5}{20} \times \frac{15}{19}\right) = \frac{150}{380}$ or 39.5% or 0.395

(c) Additional expected sales = (0.75 x 1,000) + (0.25 x 10,000) = 3,250

(d)

(i) Equally likely
(ii) Equally likely
(iii) Not equally likely

3. Comments on learners' performance

This question required students to distinguish between different types of data and to determine and apply the laws of probability. Few students attempted this question, with the average mark for this question being particularly low.

Part (a) required students to distinguish between 'continuous' and 'discrete' variables, using examples. In the main this part was well attempted, although as in previous exam sessions a good number of students were unable to adequately explain the difference between the two terms.

Part (b) required students to construct a tree diagram using the data presented in the question to show the probabilities of various outcomes. Although students generally produced a well-structured tree diagram, a number of students incorrectly calculated the probabilities associated with selecting the 'second ball' having not understood the implication of the ball not being put back in the box.

Part (c) required students to calculate expected values. In the main, this was particularly poorly answered with most students demonstrating little understanding of how to calculate expected values.

Part (d) required students to classify different outcomes as either 'equally likely' or 'not equally likely'. A large number of students were unable to classify all the outcomes correctly, demonstrating a lack of any real understanding of the terms.

4. Recommendations

Students and tutors are advised to spend a little more time conceptualising business problems and determining probabilities using tree diagrams. In addition, students should gain a better understanding of the term 'equally likely outcome' in the context of probability theory.

Examiner's tips

Familiarise yourself with what an 'equally likely' and a 'not equally likely' outcome is.

Question 8

(a) In 2015, a small retailer is considering carrying out a promotional campaign to increase sales. The probability that this promotional campaign will generates an additional 2,000 sales is 0.8 and the probability it generates an additional 40,000 sales is 0.2.
Calculate the expected sales from carrying out this promotional strategy.

(4 marks)

(b) The weight of a bar of chocolate is found to be normally distributed with a mean of 50 grams and a standard deviation of 2 grams. Calculate the probability that a randomly selected bar of chocolate weighs:
(i) less than 48 grams
(ii) more than 46 grams
(iii) between 47 grams and 49 grams

(13 marks)

(c) The weight of a bar of a different brand of chocolate is found to be normally distributed with a mean of 100 grams and a standard deviation of 4 grams. The probability that a randomly selected bar of this brand of chocolate weighing more than 110 grams is 0.621%.
(i) Sketch a standard normal distribution curve and represent this probability as an area under that normal distribution curve.
(ii) If 320 bars of this brand of chocolate were selected at random, calculate how many would weigh more than 110 grams. (Give your answer rounded up to the nearest whole bar of chocolate)

(8 marks)
Total 25 Marks

Learning Outcome 6: Apply the laws of probability
6.4 Determine the expected value of a variable
6.5 Determine probabilities using the normal distribution making use of tables
6.6 Represent normal probabilities as areas under the standard normal curve

1. Mark scheme

(a) Award 4 marks for the correct answer and up to 2 marks where the final answer is incorrect but there is evidence that the correct method has been used.
(b) Award a total of 4 marks each for parts (i) and (ii) and 5 marks for part (iii) for correctly calculating the probabilities. Method marks should be awarded (up to 3 marks for each part) where the final answer is incorrect but there is evidence that the correct method has been used.
(c) For part (i), award up to 4 marks for a correct sketch of the standard normal distribution curve, with the probability clearly shown as an area under the right hand tail of the curve.
For part (ii), award 4 marks for the correct answer, rounded up to the nearest whole bar of chocolate. Award a maximum of 3 marks if the answer has not been rounded up to the nearest bar of chocolate and up to 2 marks where the final answer is incorrect but there is evidence that the correct method has been applied.

2. Model Answer

(a) Expected sales = (0.8 x 2,000) + (0.2 x 40,000) = 9,600

(b)

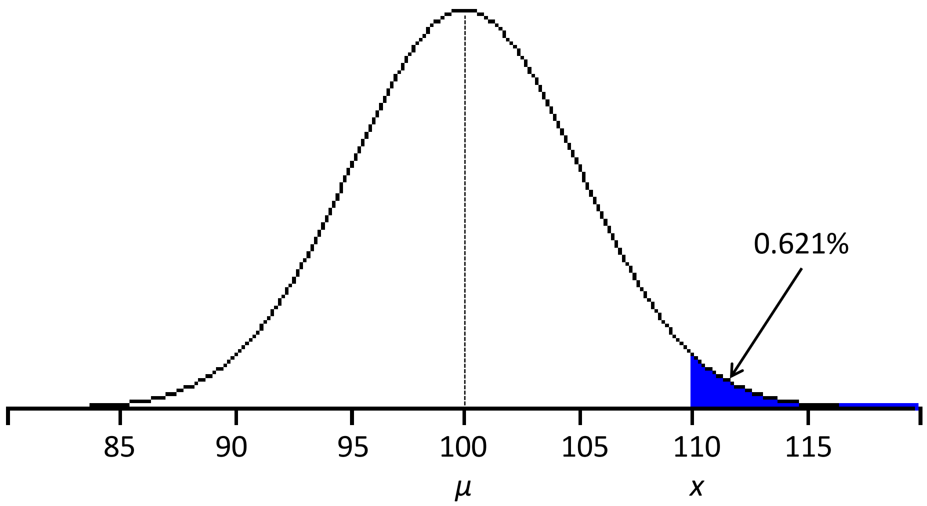
(i) $P(X < 48) = P(Z < \frac{48-50}{2}) = P(Z < -1.0) = 0.1587$ or 15.87%

(ii) $P(X > 46) = P(Z > \frac{46-50}{2}) = P(Z > -2.0) = 1 - 0.02275$
$= 0.97725$ or 97.73%

(iii) $P(47 < X < 49) = P(\frac{47-50}{2} < Z < \frac{49-50}{2}) = P(-1.5 < Z < -0.5) = 0.3085 - 0.0668$
$= 0.2417$ or 24.17%

(c)

(i)



(ii) 320 x 0.621% = 320 x 0.00621 = 1.9872 = 2 bars of chocolate

3. Comments on learners' performance

This question concerned the application of the laws of probability.

Part (a) required the calculation of expected sales from different outcomes. In the main, this question was poorly answered with most students demonstrating little understanding of how to calculate expected values. A number of students that did demonstrate an understanding of how to calculate expected sales from each outcome failed to sum the expected sales from each outcome.

Part (b) required students to determine probabilities using the normal distribution making use of tables. This part was generally answered well. As in previous years, students generally performed better when calculating probabilities to one side of the normal distribution compared with the calculation of probability within a specific range.

Part (c) required students to represent a probability graphically as areas under the normal distribution as well as interpret what this probability means in practice. This latter element was answered less well, with many students unable to interpret what the probability means in practice.

4. Recommendations

Students and tutors are advised to spend a little more time conceptualising business problems when calculating expected monetary values and determining probabilities using the normal distribution, particularly when calculating probabilities to one side of the normal distribution.

Examiner's tips

Remember to sum the expected values of each outcome when calculating the expected value of multiple outcomes.