The Association of Business Executives

NQF

Certificate

1.4 IQM

Introduction to Quantitative Methods

Thursday 11 June 2015, Morning

1. Time allowed: **3 hours**.
2. Answer **any four** questions.
3. All questions carry **25 marks**. Marks for subdivisions of questions are shown in brackets.
4. **No books, dictionaries, notes or any other written materials are allowed in this examination.**
5. **Calculators, including scientific calculators, are allowed provided they are not programmable and cannot store or recall information. All other electronic devices, including mobile phones, are not permitted.**
6. **Appropriate intermediate steps in the calculations must be shown.**
7. **Formulae** and a table of **standard normal distribution** are printed on pages **8 to 11** for the assistance of candidates. **Graph paper** is provided at the front of the answer booklet.
8. Note that £1 = 100 pence (p).
9. **Candidates who break ABE Examination Regulations will be disqualified from the examinations.**
10. Question papers must not be removed from the examination room.

Answer any four questions

Q1 **(a)** Without the use of a calculator find the value of the following, showing all steps in your calculations. (Express each answer as a fraction in its simplest form).

(i) $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

(ii) $\frac{(-2) \times (-1)}{(-2) \times (14 - (-18))}$

(iii) $\sqrt{\frac{36}{144} \times \left(\frac{1}{2} \div \frac{8}{2} \right)}$

(10 marks)

(b) A firm has estimated that the probability of a parcel being delivered on time is 0.85. Express this probability as a:

(i) Fraction in its simplest form
(ii) Percentage
(iii) Correct to 1 decimal place

(6 marks)

(c) A food manufacturing company generated an operating profit of £524,622.90 in the 2014/15 financial year.

(i) Express this operating profit correct to 3 significant figures.
(ii) If the company's operating profit was taxed at 20%, calculate the amount of tax the company would have to pay. (Give your answer to the nearest £.)

(5 marks)

(d) Convert the following numbers from standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer, to the normal decimal form:

(i) 25.8×10^{-2}
(ii) 4.26×10^3

(4 marks)
(Total 25 marks)

Q2 **(a)** A University is considering placing an advertisement in a student magazine to advertise the new Business Studies qualifications that it is offering students from September 2015. The cost of placing an advertisement in this magazine is £300 and the magazine is distributed to 5,000 students. Given this information:

(i) Calculate the cost of the advertisement per copy of the magazine distributed.
(ii) Assuming that the advertisement results in 50 students applying to study at the University, calculate the advertising cost per student application.
(iii) Calculate the expected number of student applications for every £120 spent on advertising in this particular magazine.

(10 marks)

(b) An employee is contracted to work 8 hours per day between Monday and Friday. She is paid a basic rate of £7.20 per hour worked. Any additional hours worked between Monday and Friday are paid at a higher rate of £10.80 per hour. Her basic hourly rate is doubled for every hour worked at weekends.

(i) Calculate the employee's basic weekly wage (i.e. assuming that she does not work any additional hours to those contracted).
(ii) In a particular week, the employee worked four additional hours between Monday and Friday, and six hours on Saturday. Calculate her total weekly wage for that week.
(iii) Calculate how many hours she would need to work at a weekend to receive the same increase in her basic wage as if during the week she were to work six additional hours to those contracted.

(9 marks)

(c) An American investor has purchased a Chinese company for CNY 1 million, plus a 3% sales tax. The current Chinese Yuan (CNY) to US Dollar (USD) exchange rate is:

0.16259 CNY = 1.00 USD

Calculate the total purchase cost of this investment, including the sales tax, in US Dollars. (Give your answer to the nearest Dollar.) **(6 marks)**

(Total 25 marks)

Q3 **(a)** In January 2015, a steel manufacturing company purchased new equipment costing £375,000. Calculate the value of the equipment in January 2020 if it is depreciated by:

(i) £25,000 per year using the straight line method
(ii) 10% per year using the reducing balance method

(10 marks)

(b) An investor deposits £80,000 in a bank. The bank pays interest, compounded annually, at a rate of 2.9% per year.

(i) Calculate the total interest received by the investor after five years. (Give your answer to the nearest £.)
(ii) Suppose the bank pays interest compounded half yearly instead of annually, at a rate of 1.45% per half year. Calculate how much the investment would be worth after five years. (Give your answer to the nearest £.)

(10 marks)

(c) Calculate the annual rate of compound interest that would be necessary for £80,000 to grow to £125,000 by the end of five years. (Give your answer correct to 1 decimal place.) **(5 marks)**

(Total 25 marks)

Q4 **(a)** If a paint manufacturing company's annual production costs are represented by the equation $y=0.42x+75$, where y is the company's total cost of production (in £'000) and x is the number of tins of paint produced (in '000), determine the:

(i) Company's total production cost if the number of tins of paint produced is 120,000.
(ii) Number of tins of paint produced if total production costs are £142,200.

(6 marks)

(b) Solve the following equations:

(i) $8x-24=3x+16$

(ii) $\frac{x}{2}+\frac{x}{4}=\frac{3}{5}$

(iii) $x^2-8x+15=0$, using factorisation
(iv) $x^2-5x+4.56=0$, using the quadratic formula

(15 marks)

(c) Simplify the following logarithm expression to a single log term: $\log(x-7)+\log(x)$

(4 marks)

(Total 25 marks)

Q5 **(a)** Explain the difference between 'quantitative data' and 'qualitative data', using examples. **(4 marks)**

(b) Classify the following operational data, used by a delivery company to monitor performance, as either continuous or discrete:

(i) Average number of parcels delivered per hour
(ii) Time taken to deliver ten parcels
(iii) Total weight of parcels delivered each day
(iv) Distance travelled during the day by each delivery driver

(4 marks)

(c) Given the quadratic equation $y=x^2-8x+12$:

(i) Construct a table and calculate the value of y for the following values of x: -1, 0, 1, 2, 3, 4, 5, 6, 7, 8.
(ii) Using your tabulated data in **(i)**, plot a graph of $y=x^2-8x+12$ for the values of x from $x=-1$ to $x=8$. (Use the graph paper at the front of your answer book.)

(11 marks)

(d) Using the graph of $y=x^2-8x+12$ plotted in **(c)**, find the:

(i) Values of x when $y=0$
(ii) Values of x and y when $y=x^2-8x+12$ is at its minimum
(iii) Co-ordinates at which the function $y=x^2-8x+12$ intersects the y-axis

(6 marks)
(Total 25 marks)

Q6 The following table shows the grouped frequency distribution of the height (cm) of a sample of students studying business management subjects.

Class interval of student heights (cm)	Number of students
120 to less than 130	1
130 to less than 140	3
140 to less than 150	5
150 to less than 160	16
160 to less than 170	21
170 to less than 180	28
180 to less than 190	5
190 to less than 200	1

(a) Using the grouped frequency distribution table:

(i) Determine the modal class interval of the students' heights.
(ii) Calculate the mean height of the students.

(6 marks)

(b) Using the frequency distribution data presented in the table above, construct a cumulative frequency distribution table of student heights using class intervals of 'less than 130', 'less than 140', 'less than 150', 'less than 160', 'less than 170', 'less than 180', 'less than 190' and 'less than 200'. **(4 marks)**

(c) Using the cumulative frequency distribution data constructed in **(b)**, draw a fully-labelled cumulative frequency curve (ogive) showing the height distribution of the sample of students studying business management subjects. (Use the graph paper provided at the front of your answer book.) **(6 marks)**

(d) Using your cumulative frequency curve drawn in **(c)**, determine the:

(i) Median height of students
(ii) The upper and lower quartile values of the student heights, and use these values to calculate the interquartile range

(6 marks)

(e) Explain why the 'interquartile range' is a more reliable measure of dispersion than the 'range'. **(3 marks)**

(Total 25 marks)

Q7 **(a)** Explain, using examples, what is meant by the term 'subjective probability'. **(9 marks)**

(b) A retailer sells 50 chocolate products, 20 of which are produced from South American cocoa with the remainder produced from African cocoa. Of the 20 chocolate products produced from South American cocoa, 15 contain less than 99% cocoa. Only 10 of the 50 chocolate products are made from 99% or more cocoa.

Use this information to construct a two-way contingency table and use it to calculate the probability that a chocolate product picked at random:

(i) Is produced from African cocoa
(ii) Contains less than 99% cocoa
(iii) Is produced from African cocoa **and** contains less than 99% cocoa
(iv) Is produced from African cocoa **or** contains less than 99% cocoa

(16 marks)
(Total 25 marks)

Q8 **(a)** A game at a casino involves rolling two 6-sided dice. To win, a score of less than 4 in total from both dice added together must be rolled. Calculate the:

(i) Probability of winning a game
(ii) Expected profit for the casino if 200 people each pay £1 per game to play and the prize for scoring less than 4 in total from both dice is £5. (Give your answer to the nearest £.)

(8 marks)

(b) The weight of a bag of rice is found to be normally distributed with a mean of 1,000 grams and a standard deviation of 10 grams.

(i) Find the probability that a randomly selected bag of rice weighs more than 1,030 grams.
(ii) Sketch a standard normal distribution curve and represent the probability calculated in (i) as an area under your normal distribution curve.
(iii) Find the probability that a randomly selected bag of rice weighs between 995 grams and 1030 grams.
(iv) If 2,500 bags of rice were selected at random, calculate how many bags would weigh more than 1,030 grams. (Give your answer rounded up to the nearest bag of rice.)

(17 marks)
(Total 25 marks)

End of question paper

INTRODUCTION TO QUANTITATIVE METHODS

FORMULAE FOR BUSINESS MATHEMATICS AND STATISTICS

INTEREST

The formula for calculating compound interest:

$$A = P\left(1 + \frac{r}{100}\right)^n$$

where: A = Accrued amount

P = Original principal

r = Rate of interest (for a particular time period, usually annual)

n = Number of time periods.

DEPRECIATION

- Straight-line method:

$$\text{Annual depreciation} = \frac{\text{Cost of asset}}{\text{Useful life}}$$

or $$\text{Annual depreciation} = \frac{\text{(Cost of asset) – (Value at end of useful life)}}{\text{Useful life}}$$

- Reducing balance method:

$$D = B(1 - i)^n$$

where: D = Depreciated value at the end of the nth time period

B = Original value at beginning of time period

i = Depreciation rate (as a proportion)

n = Number of time periods (normally years)

STRAIGHT LINE

A linear function is one for which, when the relationship is plotted on a graph, a straight line is obtained.

The expression of a linear function, and hence the formula of a straight line, takes the following form:

$$y = mx + c$$

Note that: c = the y intercept (the point where the line crosses the y axis)

m = the gradient (or slope) of the line

QUADRATIC EQUATION

A quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

RULES FOR LOGARITHMS

1. $\log(p \times q) = \log p + \log q$
2. $\log\left(\frac{p}{q}\right) = \log p - \log q$
3. $\log p^n = n \log p$
4. If $y = ax^n$ then $n = (\log y - \log a) \div \log x$

PROBABILITY

- Probability rules:

Probability limits:	$0 \leq P(A) \leq 1$
Total probability rule:	$\Sigma P = 1$ (for all outcomes)
For complementary events:	$P(A) + P(\bar{A}) = 1$
For two mutually exclusive events:	$P(A \text{ and } B) = 0$
For independent events:	$P(A) = P(A \mid B)$ and/or $P(B) = P(B \mid A)$

- Multiplication rules:

For independent events:	$P(A \text{ and } B) = P(A)P(B)$
For dependent events:	$P(A \text{ and } B) = P(A)P(B \mid A)$

- Additional rules:

For mutually exclusive events:	$P(A \text{ or } B) = P(A) + P(B)$
For non-mutually exclusive events:	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

- Conditional rules:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} \quad \text{and} \quad P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Expected value of variables x with associated probabilities $P(x)$ is $E(x) = \Sigma x P(x)$

STATISTICAL MEASURES

- Mean for ungrouped data: $\mu = \sum \frac{x}{N}$ and $\bar{x} = \sum \frac{x}{n}$

  where N and $\bar{x}$ are the population and sample means respectively.

- Mean for grouped data: $\mu = \Sigma mf/N$ and $\bar{x} = \Sigma mf/n$

  where m is the midpoint and f is the frequency of a class.

- Median for ungrouped data:

  = Value of the $\left(\frac{n+1}{2}\right)$th observation in a ranked data set, where the number of observations is odd and where n is the number of observations.

- Range = Largest value – Smallest value.

- Standard deviation for ungrouped data:

$$\sigma = \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{N}}{N}} \text{ and } s = \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}}$$

  where σ and s are the population and sample standard deviations respectively.

- Standard deviation for grouped data:

$$\sigma = \sqrt{\frac{\sum m^2 f - \frac{\left(\sum mf\right)^2}{N}}{N}} \text{ and } s = \sqrt{\frac{\sum m^2 f - \frac{\left(\sum mf\right)^2}{n}}{n-1}}$$

- Pearson's measure of skewness:

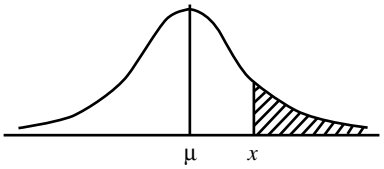
$$\text{Psk} = \frac{\textit{Mean} - \textit{Mode}}{\text{Standard deviation}} \text{ or } \frac{3(\textit{Mean} - \textit{Median})}{\text{Standard deviation}}$$

- Coefficient of variation:

$$= \frac{\text{Standard deviation}}{\text{mean}} \times \frac{100}{1}$$

STANDARD NORMAL DISTRIBUTION

The table of values of the standard normal distribution set out below provides a means of determining the probability of an observation (x) lying within specified standard deviations (σ) of the mean of the distribution (μ).



AREAS IN TAIL OF THE STANDARD NORMAL DISTRIBUTION

$\frac{(x-\mu)}{\sigma}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3874	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135									