NQF

Certificate

1.4 IQM

Introduction to Quantitative Methods

Thursday 9 June 2016, Morning

1. Time allowed: 3 hours.
2. Answer any four questions.
3. All questions carry 25 marks. Marks for subdivisions of questions are shown in brackets.
4. No books, dictionaries, notes or any other written materials are allowed in this examination.
5. Calculators, including scientific calculators, are allowed provided they are not programmable and cannot store or recall information. All other electronic devices, including mobile phones, are not permitted.
6. Appropriate intermediate steps in the calculations must be shown.
7. Formulae and a table of standard normal distribution are printed on pages 6 to 8 for the assistance of candidates. Graph paper is provided at the front of the answer booklet.
8. Note that £1 = 100 pence (p).
9. Candidates who break ABE Examination Regulations will be disqualified from the examinations.
10. Question papers must not be removed from the examination room.

Answer any four questions

Q1 **(a)** Without the use of a calculator, express each answer as a fraction in its simplest form. (You must show all steps in your calculations).

(i) $\sqrt{\frac{4}{9}} \div \left(\frac{2}{4}\right)^2$ **(4 marks)**

(ii) $\left(\frac{8}{16} \times \frac{10}{2}\right) \div 1^{0.5}$ **(4 marks)**

(iii) $\frac{-4(2-6)+(-6)-(-3)}{-4+30}$ **(4 marks)**

(b) A cocoa trading company generated sales revenue of £845,642 in the 2015/16 financial year.

(i) Express this sales revenue correct to 3 significant figures. **(2 marks)**
(ii) Express this sales revenue in standard form $A \times 10^n$ (where $1 \leq A < 10$ and n is an integer). **(2 marks)**
(iii) Calculate the value of this sales revenue in Botswana Pula (BWP), using an exchange rate of £1 = 15.52 BWP. **(2 marks)**
(iv) Calculate the percentage increase in profit between 2014/15 and 2015/16, assuming that the sales revenue of the company was £735,341 in the financial year 2014/2015. **(2 marks)**

(c) Express 0.125:

(i) correct to 2 decimal places **(1 mark)**
(ii) as a fraction in its simplest form **(2 marks)**
(iii) as a percentage **(2 marks)**

(Total 25 marks)

Q2 **(a)** An investor deposits £20,000 in a bank account that pays 6% compound interest per annum. Calculate how much interest the investor will earn after 6 years. (Give your answer to the nearest £.) **(5 marks)**

(b) Calculate the annual rate of compound interest that would be necessary for a £20,000 investment to grow to £30,000 by the end of 6 years. (Give your answer correct to 1 decimal place.) **(5 marks)**

(c) A company purchased a machine for £20,000. Calculate the value of the machine after four years if it is depreciated by:

(i) £2,000 per year using the straight line method **(5 marks)**
(ii) 10% per year using the reducing balance method **(5 marks)**

(d) Simplify the following logarithm equation to a single log term: log (x) + log (x − 12) **(5 marks)**

(Total 25 marks)

Q3 **(a)** Solve the following equations:

(i) $10x - 3 = 33 + x$ **(4 marks)**

(ii) $2x^2 - 5x + 3 = 0$, using factorisation **(4 marks)**

(iii) $x^2 + 3x + 1 = 0$, using the quadratic formula. (Give your answer correct to 2 decimal places.) **(5 marks)**

(b) For each of the following straight lines, find the values of m and c to identify the linear equation in the form $y = mx + c$:

(i) Line A passes through the point (5,8), with an intercept on the y-axis of 3 **(3 marks)**
(ii) Line B passes through the point (−4,0), with a gradient of 2 **(3 marks)**
(iii) Line C passes through the points (−2,4) and (6,8) **(6 marks)**

(Total 25 marks)

Q4 **(a)** Explain the circumstances in which a scatter diagram would be the most appropriate form of representing data graphically. **(6 marks)**

(b) The following table presents data on the number of ice cream sales and air temperature over a 15-day period in the summer of 2015:

Air temperature (°C)	Number of ice cream sales ('000)
16	1.2
14	1.0
13	0.2
24	2.2
32	3.6
15	1.1
19	1.3
14	0.4
27	2.9
21	1.9
28	3.4
19	1.5
22	1.9
14	0.6
32	3.6

(i) Draw a fully labelled scatter diagram of air temperature (on the x-axis) against ice cream sales (on the y-axis). (Use the graph paper at the front of your answer book.) **(9 marks)**

(ii) Using your scatter diagram drawn in (i), comment on the possible relationship between air temperature and ice cream sales. **(4 marks)**

(c) The demand function for ice cream (calculated from the data at air temperatures of over 15°C presented in **(b)**) is given by the equation $y = 0.1613x - 1.5019$, where y is number of ice cream sales ('000) and x is the air temperature (°C).

Using this equation, calculate the:

(i) Number of daily ice cream sales when the air temperature is 25°C. **(3 marks)**
(ii) Air temperature at which 3,000 ice creams would be sold per day. **(3 marks)**

(Total 25 marks)

Q5 **(a)** List two examples of qualitative data and two examples of quantitative data. **(4 marks)**

(b) Classify each of the following sales data as **either** continuous **or** discrete:

(i) Total number of products sold **(1 mark)**
(ii) Weight of each product sold **(1 mark)**
(iii) Time taken to sell 100 products **(1 mark)**
(iv) Number of products sold by each employee **(1 mark)**
(v) Age of each employee working in the sales team **(1 mark)**
(vi) Distance that each product sold has to be transported **(1 mark)**

(c) The following data represents the weight in kilograms (kgs) of 18 products produced by a manufacturer on a given day:

1.4	3.1	3.8	5.8	4.9	4.4
3.1	5.2	3.1	0.7	5.6	4.7
4.6	3.7	2.2	5.3	4.5	2.2

(i) Draw a fully labelled stem and leaf diagram of product weights. **(6 marks)**
(ii) Using your stem and leaf diagram, determine the:
- modal product weight **(2 marks)**
- median product weight **(2 marks)**
- range of the product weights **(2 marks)**

(iii) Comment on the distribution of the product weight data. **(3 marks)**

(Total 25 marks)

Q6 **(a)** The following data show the number of employees working at ten grocery shops:

38 41 25 35 25 20 34 21 29 12

(i) Calculate the mode, median and mean number of employees. **(6 marks)**
(ii) Calculate the range and standard deviation of the employee data. **(7 marks)**
(iii) Based on the measures of location and measures of dispersion calculated in (i) and (ii), comment on the distribution of the employee data. **(3 marks)**

(b) The expenditure of 100 customers shopping at a food store on a given day is shown in the following table:

Expenditure (£)	Frequency
0 to less than 10	10
10 to less than 20	14
20 to less than 30	28
30 to less than 40	32
40 to less than 50	11
50 to less than 60	5

Using this data, calculate the:

(i) Mean expenditure on food **(3 marks)**
(ii) Standard deviation of the food expenditure data **(6 marks)**

(Total 25 marks)

Q7 **(a)** A box contains 20 pieces of fruit, of which 15 are apples and 5 are oranges. A piece of fruit is randomly selected from the box. This piece of fruit is **not** put back in the box. A second piece of fruit is then randomly selected from the box of the remaining 19 pieces of fruit. Using this information:

(i) Draw a tree diagram to show the number of possible outcomes and their associated probabilities. **(8 marks)**

(ii) Comment on whether all the possible outcomes are equally likely or not equally likely. **(3 marks)**

(b) A production line manager carried out a random inspection of 300 products produced by two employees; 150 were produced by employee A and 150 were produced by employee B. The manager found that 30 of the products produced were of poor quality, with the remainder being of acceptable quality. Of the 150 products that were produced by employee A, 140 were found to be of acceptable quality. In contrast, 20 were found to be of poor quality out of the 150 products that were produced by employee B.

Using a contingency table or otherwise, calculate the probability that a product selected at random from these 300 products:

(i) Is of acceptable quality **(3 marks)**

(ii) Was produced by employee A **(3 marks)**

(iii) Was produced by employee A and is of poor quality **(4 marks)**

(iv) Was produced by employee B, given that it is of acceptable quality **(4 marks)**

(Total 25 marks)

Q8 **(a)** A company plans to sell ice cream at an outdoor event. If the weather is fine, then the company expects to make a profit of £6,000. However, if it rains then the event will be cancelled and the company will make a loss of £14,000. The weather forecast for the day of event is 15% possibility of rain.

Using this information, calculate the expected monetary value (EMV) in terms of the profit made by the sale of ice cream. **(5 marks)**

(b) The daily demand (kgs) for bananas in a town is normally distributed with a mean of 1,000 kgs and a standard deviation of 100 kgs.

Calculate the probability that the demand for bananas on a given day is:

(i) Less than 875 kgs **(4 marks)**

(ii) More than 1,225 kgs **(4 marks)**

(iii) Between 1,000 kgs and 1,225 kgs **(4 marks)**

(iv) Between 1,100 kgs and 1,225 kgs **(4 marks)**

(v) Between 900 kgs and 1,225 kgs **(4 marks)**

(Total 25 marks)

End of questions

Turn over for formulae and a table of standard normal distribution

FORMULAE FOR BUSINESS MATHEMATICS AND STATISTICS

INTEREST

The formula for calculating compound interest:

$$A = P\left(1 + \frac{r}{100}\right)^n$$

where: A = accrued amount
P = original principal
r = rate of interest (for a particular time period, usually annual)
n = number of time periods.

DEPRECIATION

- Straight-line method:

$$\text{Annual Depreciation} = \frac{\text{Cost of asset}}{\text{Useful life}}$$

$$\textit{or } \text{Annual Depreciation} = \frac{(\text{Cost of asset}) - (\text{Value at end of useful life})}{\text{Useful life}}$$

- Reducing balance method:

$D = B(1 - i)^n$

where: D = depreciated value at the end of the n^{th} time period
B = original value at beginning of time period
i = depreciation rate (as a proportion)
n = number of time periods (normally years).

STRAIGHT LINE

A linear function is one for which, when the relationship is plotted on a graph, a straight line is obtained.

The expression of a linear function, and hence the formula of a straight line, takes the following form:

$y = mx + c$

Note that: c = the y intercept (the point where the line crosses the y axis)
m = the gradient (or slope) of the line

QUADRATIC EQUATION

A quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

RULES FOR LOGARITHMS

1. $\log(p \times q) = \log p + \log q$
2. $\log\left(\frac{p}{q}\right) = \log p - \log q$
3. $\log p^n = n \log p$
4. if $y = ax^n$ then $n = (\log y - \log a) \div \log x$

PROBABILITY

- Probability rules:
  - Probability limits: $0 \leq P(A) \leq 1$
  - Total probability rule: $\Sigma P = 1$ (for all outcomes)
  - For complementary events: $P(A) + P(\overline{A}) = 1$
  - For two mutually exclusive events: $P(A \text{ and } B) = 0$
  - For independent events: $P(A) = P(A|B)$ and/or $P(B) = P(B|A)$
- Multiplication rules:
  - For independent events: $P(A \text{ and } B) = P(A) \times P(B)$
  - For dependent events: $P(A \text{ and } B) = P(A) \times P(B|A)$
- Additional rules:
  - For mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$
  - For non-mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- Conditional rules:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \quad \text{and} \quad P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Expected value of variables x with associated probabilities P(x) is $E(x) = \Sigma xP(x)$

STATISTICAL MEASURES

- Mean for ungrouped data:

$$\bar{x} = \frac{\Sigma x}{n}$$

- Mean for grouped data:

$$\bar{x} = \frac{\Sigma fx}{n} \quad \textit{or} \quad \bar{x} = \frac{\Sigma fx}{\Sigma f}$$

- Standard deviation for ungrouped data:

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

- Standard deviation for grouped data:

$$\sigma = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{1}{n}\left(\Sigma fx^2 - \frac{(\Sigma fx)^2}{n}\right)}$$

where: $n = \Sigma f$

- Pearson's measure of skewness:

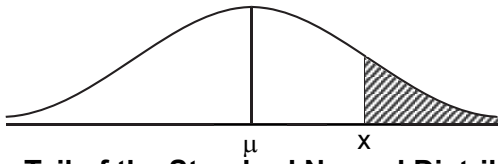
$$Psk = \frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}} \quad \textit{or} \quad \frac{3(\text{Mean} - \text{Median})}{\text{Standard deviation}}$$

- Coefficient of variation:

$$\frac{\text{Standard deviation}}{\text{Mean}} \times \frac{100}{1}$$

STANDARD NORMAL DISTRIBUTION

The table of values of the standard normal distribution set out below provides a means of determining the probability of an observation (x) lying within specified standard deviations (σ) of the mean of the distribution (μ).



Areas in Tail of the Standard Normal Distribution

$\frac{(x-\mu)}{\sigma}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3874	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135									

End of question paper