NQF

Certificate

1.4 IQM

Introduction to Quantitative Methods

Thursday 3 December 2015, Morning

1. Time allowed: 3 hours.

2. Answer any four questions.

3. All questions carry 25 marks. Marks for subdivisions of questions are shown in brackets.

4. No books, dictionaries, notes or any other written materials are allowed in this examination.

5. Calculators, including scientific calculators, are allowed provided they are not programmable and cannot store or recall information. All other electronic devices, including mobile phones, are not permitted.

6. Appropriate intermediate steps in the calculations must be shown.

7. Formulae and a table of standard normal distribution are printed on pages 7 to 10 for the assistance of candidates. Graph paper is provided at the front of the answer booklet.

8. Note that £1 = 100 pence (p).

9. Candidates who break ABE Examination Regulations will be disqualified from the examinations.

10. Question papers must not be removed from the examination room.

Answer any four questions

Q1 **(a)** Without the use of a calculator, express each of the following as a fraction in its simplest form. You must show all steps in your calculations.

(i) $\left(\frac{1}{4} \div \frac{1}{4}\right) + \left(\frac{6}{8} - \frac{1}{4}\right)$ **(4 marks)**

(ii) $\sqrt{\frac{16}{36}} \div \left(\frac{8}{15} \times \frac{5}{2}\right)$ **(4 marks)**

(iii) $\frac{(-1) \times (-4)}{(-2) \times (1 - (-3))}$ **(4 marks)**

(b) The annual salary of an office manager working in Botswana was 76,860.55 Pula in 2014. Express this salary:

(i) In standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer **(2 marks)**

(ii) Correct to 2 significant figures **(2 marks)**

(iii) Correct to 1 decimal place **(2 marks)**

(c) Convert each of the following numbers from standard form $A \times 10^n$ (where $1 \leq A < 10$ and n is an integer) to the normal decimal form:

(i) 37.5×10^{-3} **(2 marks)**

(ii) 6.56×10^3 **(2 marks)**

(d) Express 25% as a:

(i) fraction in its simplest form **(2 marks)**
(ii) decimal **(1 marks)**

(Total 25 marks)

Q2 **(a)** Use a calculator to find the value, correct to 3 decimal places, of:

(i) ln(1.5) **(2 marks)**

(ii) log(2.57) **(2 marks)**

(iii) $e^{-0.88}$ **(2 marks)**

(b) A multinational food company sells its food products in four geographical markets, namely Europe, America, Asia and Africa, in the ratio 9:6:5:4. If the value of the company's sales in Africa totalled £15,000, calculate the:

(i) Value of the company's sales in Asia. (**3 marks)**
(ii) Total value of the company's sales in all its geographical markets. **(3 marks)**
(iii) Share of its sales in America as a percentage of its sales in Europe. **(3 marks)**

(c) A student purchased an office desk at a 15% discounted price for £106.25.

(i) Calculate the original price of the office desk. **(3 marks)**
(ii) If the discounted price of the office desk included a tax (VAT) of 20%, calculate the value of the tax that the student paid. (Give your answer correct to 2 decimal places.) **(3 marks)**

(d) An American agricultural consultant visited Botswana to provide advice on crop production. During the visit, the consultant spent 39,440 Pula. Using the US Dollar (USD) to Botswana Pula (BWP) exchange rate of 1 USD = 9.86 BWP, calculate how much the consultant spent in US Dollars. **(4 marks)**

(Total 25 marks)

Q3 **(a)** At the end of 2015, a food company purchased new processing equipment costing £275,000. Calculate the value of the equipment at the end of 2020, if it is depreciated using the:

(i) Straight line method, by £25,000 per year. **(5 marks)**
(ii) Reducing balance method, by 10% per year. (Give your answer to the nearest £.) **(5 marks)**

(b) An investor deposits £240,000 in a high interest bank account over a ten-year period. Calculate the total interest received after the ten-year period if interest is compounded:

(i) Annually at a rate of 8% per annum **(5 marks)**
(ii) Quarterly at a rate of 2% per quarter (Give your answer to the nearest £.) **(5 marks)**

(c) Calculate the time required for £10,000 to earn £1,350 in interest, if invested in a bank that pays a simple interest rate of 9% per annum. **(5 marks)**

(Total 25 marks)

Q4 **(a)** The demand for a product can be represented by the equation $P = 900 - 0.3Q$, where P is the market price of the product (£ per unit) and Q is the quantity (units) demanded in a given period.

(i) From the equation, determine the co-ordinates at which the demand curve would intersect the *y*-axis if plotted on a graph. **(2 marks)**

(ii) From the equation, determine the gradient of this demand curve. **(2 marks)**

(iii) Using the equation, calculate the market price of the product if 500 units were demanded. **(3 marks)**

(iv) Using the equation, calculate the number of units demanded if the market price of the product was £600 per unit. **(3 marks)**

(b) Solve the following equations:

(i) $6x - 4 = 3x + 11$ **(2 marks)**

(ii) $\frac{x}{6} + \frac{x}{3} = \frac{12}{6}$ **(4 marks)**

(iii) $x^2 + 3x + 2 = 0$, using factorisation **(4 marks)**

(c) Simplify the following logarithm equation to a single log term: log $(x - 24)$ + log (x). **(5 marks)**

(Total 25 marks)

Q5 **(a)** Explain the difference between 'quantitative data' and 'qualitative data', giving an example of each. **(6 marks)**

(b) List two examples of 'continuous data'. **(2 marks)**

(c) A survey was carried out to determine the type of transport used by 360 students to get to college on a particular day. A total of 90 students used a car. All other students surveyed reported that they had used public transport to get to work, of which a third had used the train and two-thirds had used the bus.

(i) Present the data as a table to show the number of students that had used each type of transport to get to college. **(3 marks)**

(ii) Present the data as a fully-labelled bar chart to show the number of students that had used each type of transport. (Use the graph paper at the front of your answer book.) **(7 marks)**

(iii) Present the data as a fully-labelled pie chart to show the proportion of students that had used each type of transport. (Use the graph paper at the front of your answer book.) **(7 marks)**

(Total 25 marks)

Q6 **(a)** The following data shows the number of telephone calls received by a travel agent over a fourteen day period:

14 12 12 25 10 15 22 5 8 11 12 6 17 27

Using this data, calculate the:

(i) Mode number of phone calls **(1 mark)**
(ii) Median number of phone calls **(2 marks)**
(iii) Mean number of phone calls **(2 marks)**
(iv) Range of the data on daily phone calls **(2 marks)**
(v) Standard deviation of the data on daily phone calls **(5 marks)**
(vi) Co-efficient of variation of the data on daily phone calls **(2 marks)**

(b) The following table shows the number of times 500 customers shopped in a supermarket in a particular month:

Number of monthly shopping visits	Number of customers
1	27
2	11
3	20
4	68
5	83
6	75
7	64
8	82
9	50
10	20
	500

Using this information, calculate the:

(i) Mean number of shopping visits during the month **(4 marks)**
(ii) Standard deviation of the monthly shopping visit data **(7 marks)**

(Total 25 marks)

Q7 **(a)** Explain, using an example, what is meant by the term 'equally likely outcome'. **(5 marks)**

(b) An architect hopes to win two contracts in 2016, which are to be awarded independently of each other. The architect predicts that her chances of being awarded contract A is 30% and contract B is 15%. Contract A will generate an income of £40,000 and contract B will generate an income of £20,000.

Using this information, calculate the 'expected monetary value' (EMV) of the income to the architect from both contract A and contract B. **(4 marks)**

(c) A drinks company produces 5,000 bottles of apple juice per day, with each bottle containing 1 litre of juice. On a given day, 200 bottles were found to contain the wrong volume of juice, of which 50 bottles contained less than 1 litre.

(i) If one bottle is randomly selected from the daily production of 5,000 bottles, calculate:

- The probability that it will contain less than 1 litre of juice. **(4 marks)**
- The probability that it will contain more than 1 litre of juice. **(4 marks)**

(ii) If two bottles are randomly selected from the daily production of 5,000 bottles, calculate:

- The probability that both bottles will each contain exactly 1 litre of juice. **(4 marks)**
- The probability that both bottles will each contain less than 1 litre of juice. **(4 marks)**

(Total 25 marks)

Q8 The weight of a bag of rice produced is found to be normally distributed with a mean weight of 1,000 grams and a standard deviation of 120 grams.

(a) Calculate the probability that the weight of a bag of rice selected at random is:

(i) Less than 940 grams **(4 marks)**
(ii) More than 1,186 grams **(4 marks)**
(iii) Between 1,120 grams and 1,186 grams **(4 marks)**
(iv) Between 910 grams and 1,186 grams **(4 marks)**

(b) Sketch separate standard normal distribution curves for **(a)(iii)** and **(a)(iv)**, and represent each probability as an area under the standard normal distribution. **(6 marks)**

(c) If 425 bags of rice were selected at random, calculate how many bags would weigh more than 1,186 grams. (Give your answer rounded up to the nearest bag of rice.) **(3 marks)**

(Total 25 marks)

End of question paper

FORMULAE FOR BUSINESS MATHEMATICS AND STATISTICS

INTEREST

The formula for calculating compound interest:

$$A = P\left(1 + \frac{r}{100}\right)^n$$

where: A = accrued amount

P = original principal

r = rate of interest (for a particular time period, usually annual)

n = number of time periods.

DEPRECIATION

- Straight-line method:

$$\text{Annual Depreciation} = \frac{\text{Cost of asset}}{\text{Useful life}}$$

$$\textit{or } \text{Annual Depreciation} = \frac{(\text{Cost of asset}) - (\text{Value at end of useful life})}{\text{Useful life}}$$

- Reducing balance method:

$D = B(1 - i)^n$

where: D = depreciated value at the end of the n^{th} time period

B = original value at beginning of time period

i = depreciation rate (as a proportion)

n = number of time periods (normally years).

STRAIGHT LINE

A linear function is one for which, when the relationship is plotted on a graph, a straight line is obtained.

The expression of a linear function, and hence the formula of a straight line, takes the following form:

$y = mx + c$

Note that: c = the y intercept (the point where the line crosses the y axis)

m = the gradient (or slope) of the line

QUADRATIC EQUATION

A quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

RULES FOR LOGARITHMS

1. $log(p \times q) = log\ p + log\ q$
2. $log\left(\frac{p}{q}\right) = log\ p - log\ q$
3. $log\ p^n = n\ log\ p$
4. if $y = ax^n$ then $n = (log\ y - log\ a) \div log\ x$

PROBABILITY

- Probability rules:

Probability limits:	$0 \le P(A) \le 1$
Total probability rule:	$\Sigma P = 1$ (for all outcomes)
For complementary events:	$P(A) + P(\overline{A}) = 1$
For two mutually exclusive events:	$P(A \text{ and } B) = 0$
For independent events:	$P(A) = P(A\|B)$ and/or $P(B) = P(B\|A)$

- Multiplication rules:

For independent events:	$P(A \text{ and } B) = P(A) \times P(B)$
For dependent events:	$P(A \text{ and } B) = P(A) \times P(B\|A)$

- Additional rules:

For mutually exclusive events:	$P(A \text{ or } B) = P(A) + P(B)$
For non-mutually exclusive events:	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

- Conditional rules:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \quad \text{and} \quad P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Expected value of variables x with associated probabilities P(x) is $E(x) = \Sigma xP(x)$

STATISTICAL MEASURES

- Mean for ungrouped data: $\mu = \Sigma\frac{x}{N}$ and $\bar{x} = \Sigma\frac{x}{n}$

  where N and $\bar{x}$ are the population and sample means respectively.

- Mean for grouped data: $\mu = \Sigma\frac{mf}{N}$ and $\bar{x} = \Sigma\frac{mf}{n}$

  where m is the midpoint and f is the frequency of a class.

- Median for ungrouped data:

  = Value of the $\left(\frac{n+1}{2}\right)^{th}$ observation in a ranked data set, where the number of observations is odd and where n is the number of observations.

- Range = Largest value – Smallest value.

- Standard deviation for ungrouped data:

$$\sigma = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{N}}{N}} \text{ and } s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}}$$

  where σ and s are the population and sample standard deviations respectively.

- Standard deviation for grouped data:

$$\sigma = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma mf)^2}{N}}{N}} \text{ and } s = \sqrt{\frac{\Sigma m^2 f - \frac{(\Sigma mf)^2}{n}}{n-1}}$$

- Pearson's measure of skewness:

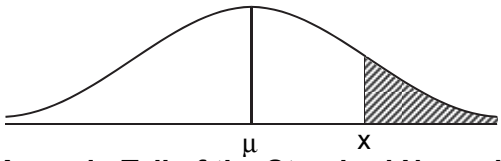
$$\text{Psk} = \frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}} \text{ or } \frac{3(\text{Mean} - \text{Median})}{\text{Standard deviation}}$$

- Coefficient of variation:

$$\frac{\text{Standard deviation}}{\text{Mean}} \times \frac{100}{1}$$

STANDARD NORMAL DISTRIBUTION

The table of values of the standard normal distribution set out below provides a means of determining the probability of an observation (x) lying within specified standard deviations (σ) of the mean of the distribution (μ).



Areas in Tail of the Standard Normal Distribution

$\frac{(x-\mu)}{\sigma}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3874	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135									

BLANK PAGE

BLANK PAGE