NQF

Certificate

1.4 IQM

Introduction to Quantitative Methods

Thursday 8 December 2016, Morning

1. Time allowed: 3 hours.
2. Answer any four questions.
3. All questions carry 25 marks. Marks for subdivisions of questions are shown in brackets.
4. No books, dictionaries, notes or any other written materials are allowed in this examination.
5. Calculators, including scientific calculators, are allowed provided they are not programmable and cannot store or recall information. All other electronic devices, including mobile phones, are not permitted.
6. Appropriate intermediate steps in the calculations must be shown.
7. Formulae and a table of standard normal distribution are printed on pages 6 to 8 for the assistance of candidates. Graph paper is provided at the front of the answer booklet.
8. Note that £1 = 100 pence (p).
9. Candidates who break ABE Examination Regulations will be disqualified from the examinations.
10. Question papers must not be removed from the examination room.

Answer any four questions

Q1 **(a)** Without the use of a calculator, find the value of the following, showing all steps in your calculations:

(i) $\frac{33}{8} \div \frac{3}{4}$ **(3 marks)**

(ii) $\frac{(-4)\times(-5)}{8-(-2)}$ **(3 marks)**

(iii) $\left(\frac{1}{2}\right)^2 \times \sqrt{\frac{9}{4}}$ **(3 marks)**

(b) Use a calculator to find the value of the following, correct to 2 decimal places:

(i) $(-0.5) \times e^{1.39}$ **(3 marks)**

(ii) $\ln(56) \div (-4)$ **(3 marks)**

(c) The sales revenue of a British food retailing company in the financial year 2013/14 was £7,755,941.56. Express this sales revenue:

(i) To the nearest £ **(2 marks)**

(ii) Correct to 4 significant figures **(2 marks)**

(iii) In standard form $A \times 10^n$ (where $1 \leq A < 10$ and n is an integer) **(2 marks)**

(d) The world price of crude oil has risen by approximately 140% over the last five years. Express this percentage rise as a:

(i) Decimal **(2 marks)**

(ii) Fraction in its simplest form **(2 marks)**

(Total 25 marks)

Q2 **(a)** A steel manufacturing company purchases new equipment costing £150,000. Calculate the value of the equipment after five years, if it is depreciated by:

(i) £18,000 per year, using the straight line method. **(5 marks)**

(ii) 18% per year, using the reducing balance method. **(5 marks)**

(b) A sum of £50,000 is to be invested over an 8-year period in two different banks that offer different interest rates and terms. Half of the amount is to be invested in Bank A, which pays simple interest at 6.0% per annum. The remainder is to be invested in Bank B, which pays compound interest at 5.9% per annum.

(i) Calculate the total interest that would be received from each bank after the 8-year investment period. (Give your answers to the nearest £.) **(10 marks)**

(ii) Calculate the annual rate of compound interest that would be necessary in order for £25,000 to grow to £40,000 by the end of 8 years. (Give your answer correct to 1 decimal place.) **(5 marks)**

(Total 25 marks)

Q3 **(a)** The demand for a product can be represented by the equation $P = 900 - 0.3Q$, where P is the market price of the product (£ per unit) and Q is the quantity (units) demanded in a given period.

(i) From the equation, determine the coordinates at which the demand curve would intersect the *y*-axis if plotted on a graph. **(2 marks)**

(ii) From the equation, determine the gradient of this demand curve. **(2 marks)**

(iii) Using the equation, calculate the market price of the product if 500 units were demanded. **(3 marks)**

(iv) Using the equation, calculate the number of units demanded if the market price of the product was £600 per unit. **(3 marks)**

(b) Solve the following equations:

(i) $6x - 4 = 3x + 11$ **(2 marks)**

(ii) $\frac{x}{6} + \frac{x}{3} = \frac{12}{6}$ **(4 marks)**

(iii) $x^2 + 3x + 2 = 0$, using factorisation **(4 marks)**

(c) Simplify the following logarithm equation to a single log term: log $(x - 24)$ + log (x) **(5 marks)**

(Total 25 marks)

Q4 **(a)** The supply curve for a particular brand of paint can be represented by the equation $P = 10 + 2Q_S$, where P is the price of the product (£ per unit) and Q_S is the quantity supplied (units) in a given period. The demand curve for the same brand of paint can be represented by the equation $P = 50 - 0.5Q_D$, where Q_D is the quantity demanded (units) in a given period.

(i) Looking at the demand equation, determine the gradient of this product's demand curve. **(2 marks)**

(ii) Looking at the supply equation, determine the coordinates at which the supply curve would intersect the *y*-axis. **(2 marks)**

(iii) Use the demand and supply equations to plot a fully labelled line graph. Use the graph paper provided at the front of your answer book. **(9 marks)**

(iv) Use your graph to determine the equilibrium price and quantity of brand of paint (i.e. the price and quantity at which the demand and supply equations intercept). **(2 marks)**

(b) The annual cost of producing the brand of paint by a paint manufacturing company can be represented by the equation $C = 30{,}000 + 0.5x$, where C is the total cost (£) of producing the branded paint and x is the quantity of tins of paint produced.

(i) Looking at the cost equation, determine the value of fixed costs (i.e. those costs that remain constant irrespective of the quantity produced). **(2 marks)**

(ii) Looking at the cost equation, determine the value of the variable costs of production per tin of paint produced. **(2 marks)**

(iii) Use the cost equation to calculate the total cost of production if the quantity of paint produced is 100,000 tins. **(3 marks)**

(iv) Use the cost equation to calculate the quantity (tins) of paint produced given that the total cost of production is £120,000. **(3 marks)**

(Total 25 marks)

Q5 **(a)** Explain the difference between 'quantitative data' and 'qualitative data', using examples. **(4 marks)**

(b) Classify the following operational data, used by a delivery company to monitor performance, as either continuous or discrete:

(i) Average number of parcels delivered per hour **(1 mark)**
(ii) Time taken to deliver ten parcels **(1 mark)**
(iii) Total weight of parcels delivered each day **(1 mark)**
(iv) Distance travelled during the day by each delivery driver **(1 mark)**

(c) Given the quadratic equation $y = x^2 - 8x + 12$:

(i) Construct a table and calculate the value of y for the following values of x: –1, 0, 1, 2, 3, 4, 5, 6, 7, 8. **(5 marks)**

(ii) Using your tabulated data in (i), plot a graph of $y = x^2 - 8x + 12$ for the values of x from $x = -1$ to $x = 8$. (Use the graph paper at the front of your answer book.) **(6 marks)**

(d) Using the graph of $y = x^2 - 8x + 12$ plotted in **(c)**, find the:

(i) Values of x when $y = 0$ **(2 marks)**

(ii) Values of x and y when $y = x^2 - 8x + 12$ is at its minimum **(2 marks)**

(iii) Coordinates at which the function $y = x^2 - 8x + 12$ intersects the y-axis **(2 marks)**

(Total 25 marks)

Q6 **(a)** Explain, using an example, what is meant by the term 'equally likely outcome'. **(5 marks)**

(b) An architect hopes to win two contracts in 2016, which are to be awarded independently of each other. The architect predicts that her chances of being awarded contract A is 30% and contract B is 15%. Contract A will generate an income of £40,000 and contract B will generate an income of £20,000.

Using this information, calculate the 'expected monetary value' of the revenue to the consulting company from both contracts A and B. **(4 marks)**

(d) A drinks company produces 5,000 bottles of apple juice per day, with each bottle containing 1 litre of juice. On a given day, 200 bottles were found to contain the wrong volume of juice, of which 50 bottles contained less than 1 litre.

(i) If one bottle is randomly selected from the daily production of 5,000 bottles, calculate:

- The probability that it will contain less than 1 litre of juice. **(4 marks)**
- The probability that it will contain more than 1 litre of juice. **(4 marks)**

(ii) If two bottles are randomly selected from the daily production of 5,000 bottles, calculate:

- The probability that both bottles will each contain exactly 1 litre of juice. **(4 marks)**
- The probability that both bottles will each contain less than 1 litre of juice. **(4 marks)**

(Total 25 marks)

Q7 **(a)** Explain the difference between 'continuous' and 'discrete' variables, giving an example of each. **(6 marks)**

(b) A box contains 20 balls, of which 15 are green balls and 5 are orange balls. A ball is selected at random from the box and its colour noted. This ball is **not** put back in the box. A second ball is then randomly selected from the box of the remaining 19 balls and its colour noted.

(i) Given this information, draw a tree diagram to show the number of possible outcomes and their associated probabilities. **(8 marks)**

(ii) Using your tree diagram or otherwise, determine the probability that the two balls selected are both:

- Green **(1 mark)**
- Orange **(1 mark)**
- Different colours **(2 marks)**

(c) The probability that a marketing campaign being run by a small food company will generate an additional 1,000 sales is 0.75 and the probability that it will generate an additional 10,000 sales is 0.25.
Calculate the additional expected sales from carrying out this marketing campaign. **(4 marks)**

(d) For the following events, classify whether all the possible outcomes are 'equally likely' or 'not equally likely':

(i) Tossing a coin **(1 mark)**
(ii) Rolling a six-sided dice **(1 mark)**
(iii) Selecting students based on nationality **(1 mark)**

(Total 25 marks)

Q8 **(a)** In 2015, a small retailer is considering carrying out a promotional campaign to increase sales. The probability that this promotional campaign will generates an additional 2,000 sales is 0.8 and the probability it generates an additional 40,000 sales is 0.2.

Calculate the expected sales from carrying out this promotional strategy. **(4 marks)**

(b) The weight of a bar of chocolate is found to be normally distributed with a mean of 50 grams and a standard deviation of 2 grams.
Calculate the probability that a randomly selected bar of chocolate weighs:

(i) Less than 48 grams **(4 marks)**
(ii) More than 46 grams **(4 marks)**
(iii) Between 47 grams and 49 grams **(5 marks)**

(c) The weight of a bar of a different brand of chocolate is found to be normally distributed with a mean of 100 grams and a standard deviation of 4 grams. The probability that a randomly selected bar of this brand of chocolate weighing more than 110 grams is 0.621%.

(i) Sketch a standard normal distribution curve and represent this probability as an area under that normal distribution curve. **(4 marks)**

(ii) If 320 bars of this brand of chocolate were selected at random, calculate how many would weigh more than 110 grams. (Give your answer rounded up to the nearest whole bar of chocolate) **(4 marks)**

(Total 25 marks)

End of questions

Turn over for formulae and a table of standard normal distribution

FORMULAE FOR BUSINESS MATHEMATICS AND STATISTICS

INTEREST

The formula for calculating compound interest:

$$A = P\left(1 + \frac{r}{100}\right)^n$$

where: A = accrued amount
P = original principal
r = rate of interest (for a particular time period, usually annual)
n = number of time periods.

DEPRECIATION

- Straight-line method:

$$\text{Annual Depreciation} = \frac{\text{Cost of asset}}{\text{Useful life}}$$

or $$\text{Annual Depreciation} = \frac{(\text{Cost of asset}) - (\text{Value at end of useful life})}{\text{Useful life}}$$

- Reducing balance method:

$D = B(1 - i)^n$

where: D = depreciated value at the end of the n^{th} time period
B = original value at beginning of time period
i = depreciation rate (as a proportion)
n = number of time periods (normally years).

STRAIGHT LINE

A linear function is one for which, when the relationship is plotted on a graph, a straight line is obtained.

The expression of a linear function, and hence the formula of a straight line, takes the following form:

$y = mx + c$

Note that: c = the y intercept (the point where the line crosses the y axis)
m = the gradient (or slope) of the line

QUADRATIC EQUATION

A quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

RULES FOR LOGARITHMS

1. $\log(p \times q) = \log p + \log q$
2. $\log\left(\frac{p}{q}\right) = \log p - \log q$
3. $\log p^n = n \log p$
4. if $y = ax^n$ then $n = (\log y - \log a) \div \log x$

PROBABILITY

- Probability rules:
  - Probability limits: $0 \leq P(A) \leq 1$
  - Total probability rule: $\Sigma P = 1$ (for all outcomes)
  - For complementary events: $P(A) + P(\overline{A}) = 1$
  - For two mutually exclusive events: $P(A \text{ and } B) = 0$
  - For independent events: $P(A) = P(A|B)$ and/or $P(B) = P(B|A)$
- Multiplication rules:
  - For independent events: $P(A \text{ and } B) = P(A) \times P(B)$
  - For dependent events: $P(A \text{ and } B) = P(A) \times P(B|A)$
- Additional rules:
  - For mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$
  - For non-mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- Conditional rules:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \text{ and } P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Expected value of variables x with associated probabilities P(x) is $E(x) = \Sigma x P(x)$

STATISTICAL MEASURES

- Mean for ungrouped data:

$$\bar{x} = \frac{\Sigma x}{n}$$

- Mean for grouped data:

$$\bar{x} = \frac{\Sigma fx}{n} \text{ or } \bar{x} = \frac{\Sigma fx}{\Sigma f}$$

- Standard deviation for ungrouped data:

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

- Standard deviation for grouped data:

$$\sigma = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{1}{n}\left(\Sigma fx^2 - \frac{(\Sigma fx)^2}{n}\right)}$$

where: $n = \sum f$

- Pearson's measure of skewness:

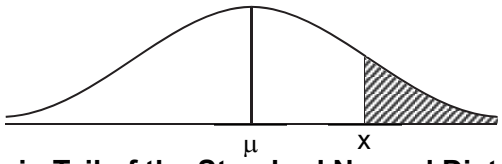
$$Psk = \frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}} \text{ or } \frac{3(\text{Mean} - \text{Median})}{\text{Standard deviation}}$$

- Coefficient of variation:

$$\frac{\text{Standard deviation}}{\text{Mean}} \times \frac{100}{1}$$

STANDARD NORMAL DISTRIBUTION

The table of values of the standard normal distribution set out below provides a means of determining the probability of an observation (x) lying within specified standard deviations (σ) of the mean of the distribution (μ).



Areas in Tail of the Standard Normal Distribution

$\frac{(x-\mu)}{\sigma}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3874	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135									

End of question paper